UNIVERSITY OF ILLINOIS BULLETIN

ISSUED TWICE A WEEK

Vol. XXXVI December 2, 1938 No. 28

[Entered as second-class matter December 11, 1912, at the post office at Urbana, Illinois, under the Act of August 24, 1912. Acceptance for mailing at the special rate of postage provided for in section 1103, Act of October 3, 1917, authorized July 31, 1918.]

AN INVESTIGATION OF RIGID FRAME BRIDGES

PART II

LABORATORY TESTS OF REINFORCED CONCRETE RIGID FRAME BRIDGES

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE PORTLAND CEMENT ASSOCIATION

BY

WILBUR M. WILSON

RALPH W. KLUGE

AND

JOHN V. COOMBE

PUBLISHED BY THE UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS

THE Engineering Experiment Station was established by act of the Board of Trustees of the University of Illinois on December 8, 1903. It is the purpose of the Station to conduct investigations and make studies of importance to the engineering, manufacturing, railway, mining, and other industrial interests of the State.

The management of the Engineering Experiment Station is vested in an Executive Staff composed of the Director and his Assistant, the Heads of the several Departments in the College of Engineering, and the Professor of Chemical Engineering. This Staff is responsible for the establishment of general policies governing the work of the Station, including the approval of material for publication. All members of the teaching staff of the College are encouraged to engage in scientific research, either directly or in coöperation with the Research Corps, composed of full-time research assistants, research graduate assistants, and special investigators.

To render the results of its scientific investigations available to the public, the Engineering Experiment Station publishes and distributes a series of bulletins. Occasionally it publishes circulars of timely interest, presenting information of importance, compiled from various sources which may not readily be accessible to the clientele of the Station, and reprints of articles appearing in the technical press written by members of the staff and others.

The volume and number at the top of the front cover page are merely arbitrary numbers and refer to the general publications of the University. *At the top of the inner title page* is given the number of the Engineering Experiment Station bulletin, circular, or reprint which should be used in referring to these publications.

For copies of publications or for other information address

THE ENGINEERING EXPERIMENT STATION,
UNIVERSITY OF ILLINOIS,
URBANA, ILLINOIS

UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

Bulletin No. 308

AN INVESTIGATION OF RIGID FRAME BRIDGES

PART II

LABORATORY TESTS OF REINFORCED CONCRETE RIGID FRAME BRIDGES

A REPORT OF AN INVESTIGATION

CONDUCTED BY

THE ENGINEERING EXPERIMENT STATION
UNIVERSITY OF ILLINOIS

IN COÖPERATION WITH

THE PORTLAND CEMENT ASSOCIATION

BY

WILBUR M. WILSON
Research Professor of Structural Engineering

RALPH W. KLUGE
Special Research Associate in Theoretical and Applied Mechanics

AND

JOHN V. COOMBE
Special Research Assistant in Civil Engineering

PUBLISHED BY THE UNIVERSITY OF ILLINOIS
ENGINEERING EXPERIMENT STATION

Price: Eighty-five Cents

4000—11-38—15582-S

UNIVERSITY
OF ILLINOIS
:: PRESS ::

CONTENTS

LIST OF FIGURES

LIST OF TABLES

AN INVESTIGATION OF RIGID FRAME BRIDGES

PART II

LABORATORY TESTS OF REINFORCED CONCRETE RIGID FRAME BRIDGES

I. Introduction

1. *Object and Scope of Investigation.*—The rigid frame bridge, although not a new type of structure, has recently come into general use, particularly in highway construction, because of its adaptability to grade separations. Its flat deck and small crown thickness result in a wide underneath clearance and a small difference in elevation of the two roadways being separated.

The structure can be analyzed algebraically in the sense that the reaction components at the bases of the vertical legs can be determined on the basis of an assumed relation between stress and strain. But neither the resistance of concrete at a re-entrant angle, nor the intensity of the stress at such angles due to given external forces is known. When an unsymmetrical load is applied to a rigid frame, the latter, if free, will sway, and the reaction components at the bases due to a load on the deck depend upon the resistance against sway that is provided by outside forces. In view of these considerations, not only were tests made to determine the action of the structure when subjected to various loads and abutment displacements in order to compare the observed action with the action anticipated from a study of the elastic properties of the structure, but observations were also made to determine the magnitude of the sway when the structure was free. The force necessary to prevent sway was measured also. The specimen was finally loaded to destruction to determine the manner of failure, and to compare the unit stress developed by the concrete in the structure with that developed by the same concrete in 6-in. x 12-in. control cylinders.

2. *Acknowledgments.*—The tests described in this bulletin are a part of the investigation resulting from a coöperative agreement between the Engineering Experiment Station of the University of Illinois, of which Dean M. L. Enger is the Director, and the Portland Cement Association, of which Frank T. Sheets is the President. The authors were assisted in planning the tests by A. J. Boase, Manager of the Structural and Technical Bureau of the Portland Cement Asso-

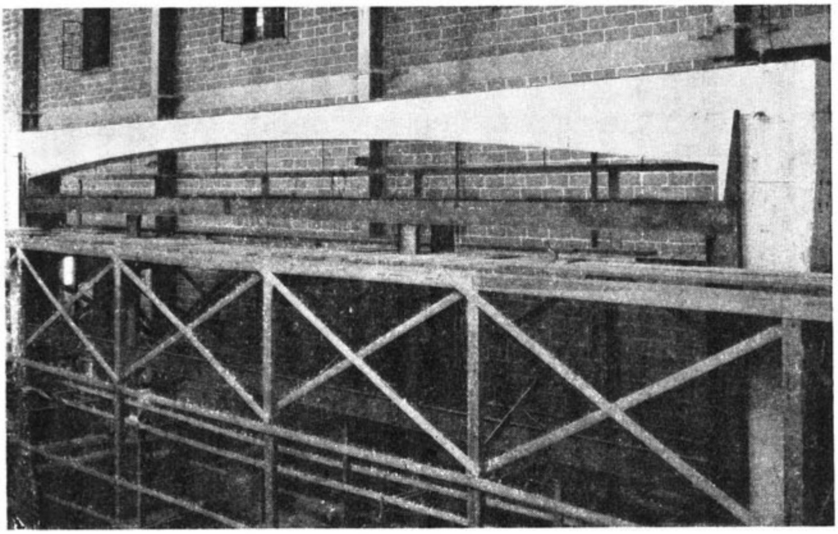


FIG. 1. REINFORCED CONCRETE RIGID FRAME BRIDGE WITH APPARATUS FOR TESTING

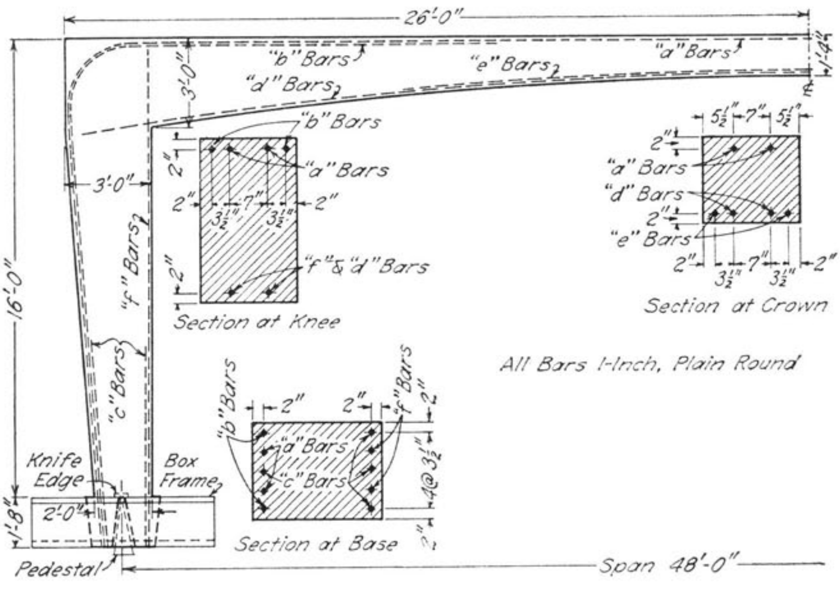


FIG. 2. DETAILS OF RIGID FRAME BRIDGE

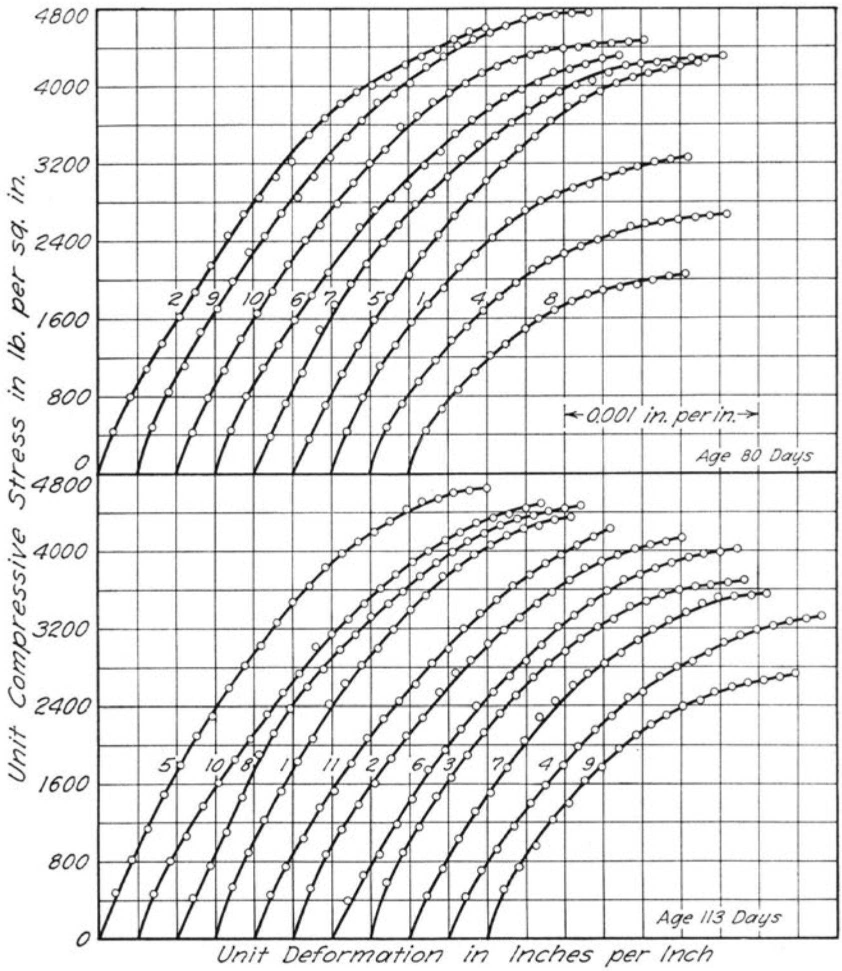


FIG. 3. STRESS-STRAIN DIAGRAMS FOR CONTROL CYLINDERS; SPECIMEN 1

ciation. The tests were made in the Arthur Newell Talbot Laboratory of the University of Illinois, and the direct expenses of the investigation were paid from funds provided by the Portland Cement Association.

II. DESCRIPTION OF SPECIMENS AND APPARATUS

3. *Description of Specimens.*—The first specimen tested, shown in Fig. 1, was designed as a highway bridge one and one-half feet wide, with a span of 48 feet center to center of the bases of the vertical legs

TABLE 1
PHYSICAL PROPERTIES OF CONCRETE
From tests of 6-in. × 12-in. control cylinders—Specimen 1

Cylinders 80 Days Old			Cylinders 113 Days Old		
Cylinder No.	Ultimate Strength lb. per sq. in.	Modulus of Elasticity in 10^6 lb. per sq. in.	Cylinder No.	Ultimate Strength lb. per sq. in.	Modulus of Elasticity in 10^6 lb. per sq. in.
1	3250	4.5	1	4360	5.4
2	4610	4.8	2	4150	4.8
4	2688	4.4	3	3710	5.7
5	4204	4.0	4	3310	4.1
6	4330	4.6	5	4630	4.6
7	4340	4.3	6	4030	3.7
8	2060	3.8	7	3550	4.2
9	4775	5.3	8	4470	4.5
10	4470	4.6	9	2740	4.7
Av.	3860	4.5	10	4480	4.9
			11	4300	4.6
			Av.	3980	4.7
			Average of all......	3920	4.6

and a height of 16 feet from base to top of deck. The proportions of the structure were similar to those used in current practice. The thicknesses at the crown, the knee, and the bottom of the vertical leg were 1 ft. 4 in., 3 ft. 0 in. and 2 ft. 0 in., respectively. Plain one-inch round reinforcing bars of steel having a yield point of 47 000 lb. per sq. in. and an ultimate strength of 80 000 lb. per sq. in. were used throughout. Details of the structure are shown in Fig. 2. A 1:3.25:4 mix, by weight, was used with a 1.2 water-cement ratio, designed to give a strength of 3500 lb. per sq. in. at 28 days. The concrete was mixed in one-bag batches, the quantities of aggregate being determined by weight and corrected for moisture content. Twenty control cylinders were made, one from each of twenty batches. The physical properties of the concrete as determined from tests of 6-in. x 12-in. control cylinders are given in Table 1, and the stress–strain diagrams for the cylinders are given in Fig. 3. The structure remained in the steel form for 12 days, then it was stripped and cured in the laboratory, where the air was dry and the temperature was approximately 80 deg. F. Testing began when the structure was 26 days old.

The specimen was analyzed by the elastic theory and designed for an H-20 truck load, a temperature change of ± 45 deg. F., and a shrinkage coefficient of 0.0003. Analyses were made for the structure with the bases both hinged and fixed, and the structure was designed to withstand the maximum stress for either condition. Earth pressure on the vertical leg was disregarded. The maximum stresses were

TABLE 2

INFLUENCE ORDINATES FOR REACTION COMPONENTS AT EAST BASE

By the Elastic Theory
$E = 3\ 000\ 000$ lb. per sq. in.
Structure free to sway

Load* Point	Bases Fixed			Bases Hinged	
	M ft. lb.	H lb.	V lb.	H lb.	V lb.
E5	0.1318	0.1805	0.9469	0.1223	0.9167
E4	0.3157	0.3481	0.8870	0.2391	0.8333
E3	0.7900	0.5026	0.8137	0.3453	0.7500
E2	1.5666	0.6324	0.7240	0.4328	0.6667
E1	2.5606	0.7211	0.6175	0.4917	0.5833
Center	3.5411	0.7530	0.5000	0.5128	0.5000
W1	4.2006	0.7211	0.3825	0.4917	0.4167
W2	4.3186	0.6324	0.2760	0.4328	0.3333
W3	3.8476	0.5026	0.1863	0.3453	0.2500
W4	2.8917	0.3481	0.1130	0.2391	0.1667
W5	1.5830	0.1805	0.0531	0.1223	0.0833

*For location of load points see Fig. 28.

TABLE 3

ELASTIC CONSTANTS

By the Elastic Theory
$E = 3\ 000\ 000$ lb. per sq. in.
Moments, thrusts, and shears are for the full width of the structure, 18-in.

Movement at Base	Reactions at East Base				
	Bases Fixed			Bases Hinged	
	M ft. lb.	H lb.	V lb.	H lb.	V lb.
Spread of 0.10 in.	−45 500	−3315	0	−230	0
Settlement of east base 0.10 in.	+6 450	0	+269	0	0
Rotation of east base $\theta = +0.001$ rad.	+104 800	+5460	−775	...	...
Rotation of west base $\theta = +0.001$ rad.	+67 500	+5460	+775	...	...

limited to 20 000 lb. per sq. in. for steel and 1200 lb. per sq. in. for concrete. E was assumed as 3 000 000 lb. per sq. in., and $n = 10$. The influence ordinates for the reaction components, given in Table 2, and the elastic constants, given in Table 3, were determined by the elastic theory.

The moment due to each of the various causes and the resultant moment and the corresponding unit stresses are given in Table 4. The live load upon which these values are based consists of two concen-

TABLE 4

MOMENTS AND FORCES CONTROLLING DESIGN

By the Elastic Theory
$E = 3\ 000\ 000$ lb. per sq. in.
Structure free to sway
Moment, thrust and shear are for the full width of the structure, 18 in.

		Crown			Knee		Base of Vertical Leg		
		Moment 10^3 lb. per sq. in.	Thrust lb.	Shear lb.	Moment 10^3 lb. per sq. in.	Vertical Thrust lb.	Moment 10^3 lb. per sq. in.	Vertical Thrust lb.	Shear lb.
Bases Fixed	Dead load	17.4	8250	0	−85.5	9 070	38.4	17 200	8 250
	Live load	28.4	5920	3710	−63.6	5 830	34.9	3 060	5 920
	Temperature	± 8.1	∓5030	....	± 3.9	......	±69.0	......	∓5 030
	Shrinkage	+ 9.2	−5710	....	+ 4.5	......	−78.5	......	−5 710
	Resultant	63.1	8460	3710	−153.0	14 900	142.3 −109.1	20 260	19 200
Bases Hinged	Dead load	19.1	5640	0	−84.0	9 070	....	17 200	5 640
	Live load	29.4	4020	3810	−60.5	4 830	....	4 830	4 020
	Temperature	±5.5	∓355	....	± 5.2	......	....	......	∓355
	Shrinkage	+6.1	−395	....	+ 5.7	......	....	......	−395
	Resultant	60.0	9295	3810	−149.7	13 900	....	......	......

Maximum Unit Stresses, lb. per sq. in.

	Crown	Knee	Base of Leg
Concrete	1 160	670	1 030
Steel	18 600	16 600	20 000

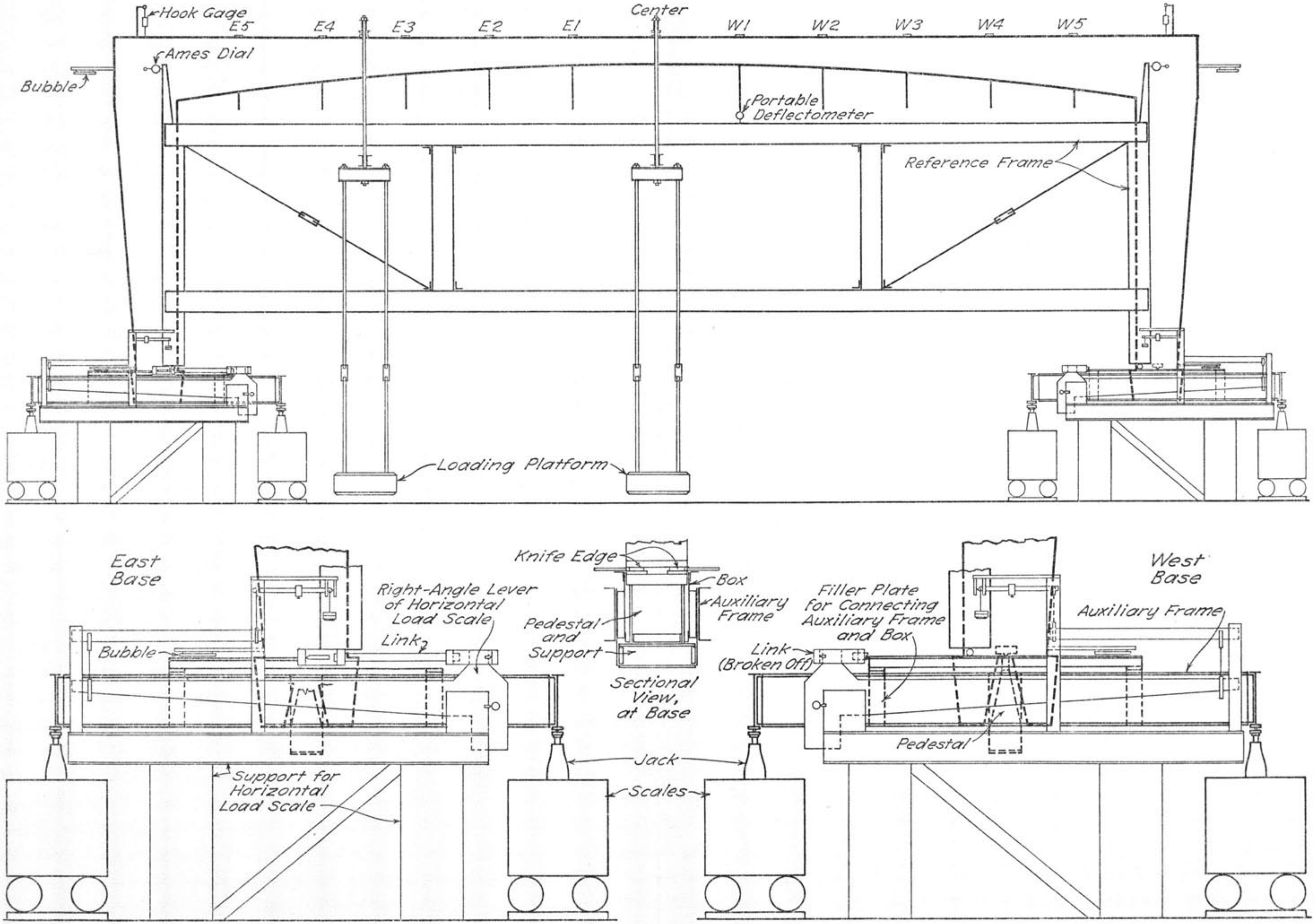


FIG. 4. GENERAL VIEW OF APPARATUS

This page is intentionally blank.

trated loads, 6900 lb. and 1720 lb., spaced 14 feet apart, and located so as to produce a maximum stress at the section considered.

The preceding description applies to the first structure that was built, which is designated as Specimen 1. A second structure was built and tested, which is designated as Specimen 2. It is similar to Specimen 1 except that the deck and legs contained shear reinforcement consisting of rectangular loops of ½-in. round bars enclosing the longitudinal reinforcing and spaced on 12 in. centers.

The following sign convention is used throughout in this report: A positive vertical reaction produces compression in the legs of the structure, positive horizontal thrust produces compression in the deck, and positive moment produces tension at the intrados.

4. *Description of Apparatus.*—Figure 4 is a general view of the specimen and apparatus. The specimen was supported at each base by a steel frame made up of an inner box, into which the lower end of the structure was poured, and an outer auxiliary frame mounted on jacks supported on scales. The supporting frame was designed so that the structure could be tested with either fixed or hinged bases. For the fixed condition, the box and auxiliary frame were fastened together with bolts and filler plates, as shown in Fig. 4. For the hinged condition, the bolts and filler plates were removed, permitting rotation of the inner box about a knife edge embedded in the concrete at the theoretical base of the structure. The knife-edge bearing block was fastened on the top of a tapered pedestal which was suspended from the auxiliary frame, as shown in detail in the figure. The pedestal was clear of the box so that any angular movement of the box, which was integral with the base of the structure, was independent of the auxiliary frame.

The weighing apparatus consisted of four vertical-load scales and four horizontal-load scales, all of which had been designed and built for tests of concrete arch bridges.* The horizontal-load scales, one on each side of the structure at each base, were anchored to the laboratory floor. They received the horizontal component of the thrust through adjustable links which connected the right-angle lever of the scales to embedded plates projecting from the face of the concrete at the bases of the specimen, as shown in Fig. 4. The knife-edge, which is the only contact between the link and the projecting plate, was accurately placed at the theoretical base of the vertical leg. The two vertical-load scales for each base were spaced 12 feet apart, and were supported on 10-inch steel rollers carefully machined and bearing on

*University of Illinois Engineering Experiment Station, Bulletin 269.

carefully leveled and machined tracks, so that resistance to horizontal movement was negligible. They received the vertical reaction through knife edges fastened to the supporting frame.

The bases of the structure could be given any one of the three components of movement (x, y, or Θ) independently of the others, and each could be measured. Vertical translation was produced by raising or lowering all four jacks under a base an equal amount. This movement was measured by two hook gages, one above each base, connected to a pipe line laid along the top of the deck. Horizontal translation was produced by turning the rods of the links, the ends of which were threaded right and left hand. This horizontal movement of the bases relative to each other was measured by means of a pair of Ames dials, one on each side, fastened to the west end of a reference frame shown in Fig. 4. This frame was pin-connected to the east base of the structure, and supported on a roller at the west base. The rotation of a base was produced by raising the inside (or outside) set of jacks and lowering the opposite set an equal amount. The angular position of the base was indicated by a sensitive calibrated bubble fastened to a bar embedded in the concrete at the base.

Vertical movement of points on the deck was measured with respect to the reference frame previously mentioned. This frame consisted of two parallel trusses, one on each side of the structure, built of 12-in. J and L junior beams and connected with diagonal cross bracing and battens. The battens along the top were spaced so as to be directly below vertical rods projecting from the lower side of the deck under each load point. Deflections were measured by a portable deflectometer inserted between the ends of the rods and the battens.

The reference frame also served as a means of measuring the sway of the structure due to unsymmetrical loading. Ames dials fastened at the top corners indicated the horizontal movement of these points relative to the bases.

Sway was prevented for some tests; for others, a predetermined sway was produced. This was accomplished by means of a horizontal strut with one end bearing on a building column of the laboratory and the other end bearing on the west vertical leg of the specimen in line with the intersection of the axes of the deck and the vertical leg. A small jack at the end of the strut was used to produce sway when desired.

The loading equipment was in two parts. One part, used in the live-load tests, consisted of two loading platforms with their loading beams and suspender rods. These are shown in the figure. The weight of this equipment, platform, suspender rods and loading beam, constituted one live load (wheel loads of 1720 and 6900 lbs., respectively) located as shown. Additional live load was obtained by adding blocks of known weight.

A steel plate grouted to the top of the deck at each load point had a steel ball embedded in the top at the point of application of the load. A small steel block attached to the lower side of the loading beam contained a depression that fitted over the top of the ball, thus assuring the accurate placement of the load. The suspender rods were fitted with turnbuckles for adjusting their length. When the structure was not loaded, the rods were extended so that, with the loading platform resting on the floor of the laboratory, the loading beam was clear of the deck and its weight was carried by the suspender rods acting as struts. To load the structure, the rods were shortened until the loading beam came into contact with the structure. The beams were then properly located relative to the steel balls, and the rods were further shortened until the platform swung clear of the floor, thus transmitting the load to the deck.

The second part of the loading equipment was the unit load of 2000 lb. used for influence-ordinate tests. This load consisted of two steel castings suspended from a loading beam in such a manner that the whole would be in equilibrium when set upon the deck, the only contact being the steel ball embedded in the steel plate grouted on the top of the deck at the load point.

III. Description of Tests

5. *Elastic Properties of Specimen.*—The elastic properties of the specimen were determined experimentally first with the bases fixed and then with them hinged. Most of the tests involved the displacement of one base with the other fixed, and with the structure free to sway. Some tests were made, however, in which sway was prevented when one base was moved vertically relative to the other. Two tests were made for which sway was produced by an external longitudinal force applied to the end of the deck. All of the tests included measurement of the vertical movement of points on the deck as well as the measurement of the changes in the reaction components accompanying the displacement of the base.

The first of these tests consisted of measuring the changes in the reaction components at the bases when the west base was given a

TABLE 5
REACTION COMPONENTS DUE TO CHANGE IN SPAN; BASES FIXED, SPECIMEN FREE TO SWAY
Specimen 1

	Change in X in.	Change in Thrust lb.			Change in Vertical Reaction lb.						Change in Moment 1000 ft. lb.		
					East Base			West Base					
		East Base	West Base	Average	Outside Scale	Inside Scale	Total	Outside Scale	Inside Scale	Total	East Base	West Base	Average
West Base Moved	+0.05 to −0.05	+3515	+3502	+3508	+3626	−3456	+170	+4097	−4266	−169	+42.4	+50.1	+46.3
	−0.05 to +0.05	−3755	−3760	−3758	−3874	+3678	−196	−4425	+4602	+177	−45.3	−54.1	−49.7
	Average $\Delta X = +0.10$	−3635	−3631	−3633	−3750	+3567	−183	−4261	+4434	+173	−43.9	−52.1	−48.0
East Base Moved	−0.05 to +0.05	−3965	−3909	−3937	−4118	+3920	−198	−4635	+4839	+204	−48.2	−56.8	−52.5
	+0.05 to −0.05	+3864	+3807	+3836	+4006	−3810	+196	+4526	−4729	−203	+46.8	+55.5	+51.2
	Average $\Delta X = +0.10$	−3915	−3858	−3887	−4062	+3865	−197	−4580	+4784	+204	−47.5	−56.1	−51.8
	Average for two tests, $\Delta X = 0.10$			−3760			−190			+189	−45.7	−54.1	−49.9

TABLE 6

REACTION COMPONENTS DUE TO ROTATION OF ONE BASE WITH THE OTHER BASE FIXED, SPECIMEN FREE TO SWAY

Specimen 1

	Change in 0.001 Radians	Change in Horizontal Reaction, lb			Change in Vertical Reaction, lb.						Change in Moment 1000 ft. lb.	
					East Base			West Base				
		East Base	West Base	Average	Outside Scale	Inside Scale	Total	Outside Scale	Inside Scale	Total	East Base	West Base
West Base Rotated	0.5117, top tipped in	+3455	+3418	+3437	+3149	−2406	+743	+5320	−6072	−752	+33.3	+68.3
	0.5117, top tipped out	−3510	−3473	−3497	−3174	+2419	−755	−5387	+6131	+744	−33.5	−69.0
	Average per 0.001 radian, top tipped in			+6775			−1464			+1462	+65.3	+134.2
East Base Rotated	0.5102, top tipped in	+2803	+2807	+2805	+4051	−4413	−362	+2924	−2550	+374	+50.7	+32.8
	0.5102, top tipped out	−2810	−2810	−2810	−4027	+4388	+361	−2926	+2558	−368	−50.4	−32.9
	Average per 0.001 radian, top tipped in			+5504			−710			+727	+99.1	+64.4

horizontal movement of 0.10 in. without rotation or vertical translation, and with the east base fixed. Preliminary to the test, the west base was moved out 0.05 in. This was the starting position for the test, the position for which the basic readings were recorded. These included the readings of the vertical-load and horizontal-load scales, the Ames dials indicating the span, the hook gages indicating the relative vertical positions of the two bases, the bubbles indicating the angular positions of the bases, and the deflectometer indicating the vertical positions of points on the deck. The west base was then moved inward 0.10 in. without rotation or vertical movement, and with the east base fixed, and a second set of readings was recorded. The west base was then returned to the starting position and a third set of readings was recorded. There were, therefore, two complete tests, one giving the changes due to a decrease, and the other those due to an increase in span. And in each instance the movement was from a condition one side of normal to another an equal amount the other side of normal. The changes in the reaction components due to a change in span, determined by the method just described, are given in the upper part of Table 5. A similar test was made in which the east base was moved and the west base was fixed. The results of this test are given in the lower part of the same table, and the average of the two is given in the last line of the table.

Tests similar to the ones described were made to determine the change in the reaction components at the bases due to rotating one base without translation and with the other base fixed. The magnitude of the rotation was measured with a calibrated bubble, and the rotation for which the change in the reaction components was measured was from a position one side of normal to a position an equal angle the other side of normal. The total rotation was approximately 0.0005 radian. There were two tests; for one the west base was rotated with the east one fixed; for the other the east base was rotated with the west one fixed, the specimen being free to sway for both tests. The results are given in Table 6.

In the tests to determine the reaction components due to a settlement of one base, the west base was raised approximately 0.10 in. above its normal position for the basic readings. It was then lowered approximately 0.2 in. with the east base fixed, for the second set of readings, and then returned to its original position for the final readings. The test was repeated, raising and lowering the east base. The results are given in Table 7. A second test was made in which sway

TABLE 7

REACTION COMPONENTS DUE TO SETTLEMENT; BASES FIXED, SPECIMEN FREE TO SWAY

Specimen 1

	Vertical Movement in.	Change in Horizontal Reaction, lb.			Change in Vertical Reaction lb.						Change in Moment 1000 ft. lb.		
					East Base			West Base					
		East Base	West Base	Average	Outside Scale	Inside Scale	Total	Outside Scale	Inside Scale	Total	East Base	West Base	Average
West Base Moved	0.1920 down	+393	+390	+392	−649	+1426	+777	+1630	−2428	−798	−12.4	+24.3	
	0.1920 up	−419	−410	−415	+649	−1417	−768	−1645	+2441	+796	+12.4	−24.5	
	Average per 0.10 in. down			+210			+403			−415	− 6.5	+12.7	9.6
East Base Moved	0.1938 down	−389	−413	−401	+621	−1415	−794	−1652	+2410	+758	+12.2	−24.0	
	0.1938 up	+422	+375	+399	−649	+1443	+794	+1635	−2397	−762	−12.5	+24.2	
	Average per 0.10 in. down			−206			−410			+392	+ 6.4	−12.4	9.4

TABLE 8

RELATION BETWEEN REACTION COMPONENTS AND LONGITUDINAL MOVEMENT OF DECK; BASES FIXED

Movement, in.	Change in Horizontal Thrust, lb.			Change in Vertical Reaction, lb.		Change in Moment, ft. lb.	
	East Base	West Base	Thrust at End of Deck	East Base	West Base	East Base	West Base
Sway Prevented; West Base Moved Vertically. Specimen 1							
West base down 0.074	+ 903	−1259	2162	+467	−470	+ 2 238	− 6 420
West base up 0.074	− 940	+1227	2167	−494	+499	− 2 324	+ 5 302
Average west base down 0.10	+1245	−1680	2925	+649	−655	+ 3 082	− 7 920
Sway Produced by Longitudinal Force at End of Deck. Specimen 1							
Deck moved east 0.011	+1113	−1897	3010	+316	−328	+ 9 456	−19 175
Deck moved west 0.011	−1198	+1807	3005	−301	+313	−11 056	+18 270
Average deck moved 0.01	+1051	−1683	2734	+280	−274	+ 9 324	−17 020
Sway Produced by Longitudinal Force at End of Deck. Specimen 2							
Deck moved east 0.0142	+1280	−1917	3197	+394	−400	+10 295	−18 240
Deck moved west 0.0142	−1302	+1898	3200	−396	+400	−10 654	+17 940
Average deck moved east 0.01	+ 909	−1344	2253	+278	−282	+ 7 365	−12 730

TABLE 9
EXPERIMENTAL ELASTIC CONSTANTS; SUMMARY OF RESULTS
Specimen 1
Bases Fixed. Specimen Free to Sway
Values are for full width of specimen, 18-in.

Movement of Base		Change in Reaction Components		
		H lb.	*V* lb.	*M* ft. lb.
Spread 0.10 in.		3760 *3315*	178 *0*	−45 910 *−45 500*
Settlement 0.10 in.		208 *0*	405 *269*	± 9 500 *± 6 450*
Rotation 0.001 rad. top tipped in	Near end	6140 *5460*	1090 *775*	+116 650 *+104 800*
	Far end	6140 *5460*	1090 *775*	+64 850 *+67 500*

was prevented by introducing a horizontal force at the west end of the deck 14.5 ft. above the bases. The results are given in Table 8.

A test was also made to determine the reaction components at the bases when, with both bases fixed, sway was produced by applying a horizontal force to the west end of the deck 14.5 ft. above the bases. The results of this test are also given in Table 8.

The results of the tests to determine the elastic constants for Specimen 1 are summarized in Table 9. The values are comparable with those computed by the elastic theory, given in Table 3. The values given in Table 9 are based upon those given in Tables 5, 6 and 7, corresponding quantities for the two ends being averaged. The corresponding values determined by the elastic theory and taken from Table 3 are given in Table 9 in italics. Although the average of the experimental values and the values determined by the elastic theory are in fair agreement in many instances, corresponding experimental values for the two ends, which for a symmetrical structure are equal, differed greatly in some instances.

The results of tests on Specimen 1 to determine the elastic constants when the bases were hinged are given in Table 10, and the relation between reaction components and longitudinal movement of the deck are given in Table 11.

One of the interesting results of the tests of Specimen 1 is the difference between reaction components at the two ends of the specimen which, for a symmetrical structure,* are alike. For a change

*Symmetrical in physical properties as well as in geometrical outline.

TABLE 10

HORIZONTAL THRUST DUE TO CHANGE IN SPAN; BASES HINGED, SPECIMEN FREE TO SWAY

Specimen 1

Values are for full width of specimen, 18 in.

Movement of Base in.		Change in Horizontal Thrust lb.		
		East Base	West Base	Average
First Test	East base moved from 0.10 out to 0.10 in.	+740	+655	+698
	East base moved from 0.10 in to 0.10 out	−778	−673	−726
Second Test	East base moved from 0.10 in to 0.10 out	−770	−676	−723
	East base moved from 0.10 out to 0.10 in	+758	+653	+706
Third Test	West base moved from 0.10 out to 0.10 in	+691	+680	+686
	West base moved from 0.10 in to 0.10 out	−713	−743	−728
Average of all tests on uncracked deck on basis of span decreased 0.10		.....	.....	+356
Fourth Test Deck Cracked	West base moved from normal to 0.10 out	−322	−358	−340
	West base moved from 0.10 out to normal	+340	+383	+362
Average of all tests on cracked deck on basis of span decreased 0.10		.....	.....	+351
By elastic theory on basis of span decreased 0.10		.....	.....	+230

in span the moment is about 20 per cent greater at the east than at the west base; and the vertical reaction, which is zero for a symmetrical structure, is 190 lb. For the rotation of a base, the rotation of the west base produces larger reaction components than an equal rotation of the east base. The difference is 23 per cent for the horizontal thrust, 100 per cent for the vertical reaction, 35 per cent for the moment at the end rotated, and approximately zero for the moment at the fixed end. For the settlement of the base the horizontal reaction, which is zero for a symmetrical structure, is one-half as great as the vertical reaction; and the moment is almost twice as great at the west as at the east base.

Because the corresponding constants for the two ends of Specimen 1 differed materially, Specimen 2, which was constructed primarily for use in tests to determine the effect of time yield in concrete upon the action of a rigid frame bridge, was also tested for the elastic constants. The detailed report of these tests, given in Table 12, indicates that in each instance the two movements of a base, equal in magnitude but opposite in direction, were accompanied by changes in the reaction components that were nearly equal in magnitude.

TABLE 11

RELATION BETWEEN REACTION COMPONENTS AND LONGITUDINAL MOVEMENT OF DECK; BASES HINGED

Values are for full width of specimen, 18 in.

Movement, in.	Change in Horizontal Reaction, lb.			Change in Vertical Reaction, lb.		
	East Base	West Base	Thrust at End of Deck	East Base	West Base	Average
Sway Prevented; East Base Moved Vertically						
East base moved upward from normal 0.049	+504	−562	1066	+325	−318	322
East base moved downward 0.048 to normal	−507	+556	1063	−326	+315	321
Average on basis that east base moved upward 0.10			2172			656
Sway Produced by Longitudinal Force at End of Deck						
Deck moved east 0.021 from normal	+802	−832	1634	+494	−479	486
Deck moved west 0.022 to normal	−843	+903	1746	−528	+512	520
Average on basis that deck moved east 0.10	+3825	−4035	7860	+2375	−2305	2340

TABLE 12
EXPERIMENTAL ELASTIC CONSTANTS; SUMMARY OF RESULTS
Bases Fixed. Specimen 2.
Values are for the full width of the structure, 18 in.

	Movement of Base in. or 0.001 rad.	East Base			West Base		
		M ft. lb.	H lb.	V lb.	M ft. lb.	H lb.	V lb.
East End Moved	$\Delta\theta = -0.453$	−38 250	−1924	+421	−18 140	−1926	−425
	$\Delta\theta = +0.453$	+39 150	+1942	−415	+18 560	+1945	+428
	Average	−38 700	−1933	−418	−18 350	−1936	−427
	$\Delta y = +0.1947$	−14 480	+ 75	+679	+16 970	+ 72	−659
	$\Delta y = -0.1947$	+14 380	− 106	−681	−17 150	− 109	+663
	Average	−14 430	+ 91	+680	+17 060	+ 90	−661
	$\Delta x = -0.10$	+34 850	+2885	+ 33	+35 880	+2729	− 15
	$\Delta x = +0.10$	−35 860	−2974	− 39	−37 090	−2828	+ 16
	Average	+35 360	+2930	+ 36	+36 490	+2780	− 16
West End Moved	$\Delta\theta = -0.461$	−17 070	−1928	−477	−39 960	−1899	+472
	$\Delta\theta = +0.461$	+16 960	+1902	+474	+39 080	+1875	−468
	Average	−17 020	−1915	−476	−39 520	−1887	+470
	$\Delta y = +0.1959$	+15 150	− 40	−645	−16 030	− 35	+676
	$\Delta y = -0.1959$	−15 450	− 16	+637	+15 380	− 17	−667
	Average	+15 300	− 12	−641	−15 700	− 9	+671
	$\Delta x = -0.10$	+39 850	+3117	+ 49	+41 390	+3214	− 46
	$\Delta x = +0.10$	−40 580	−3161	− 51	−42 110	−3257	+ 40
	Average	+40 120	+3139	+ 50	+41 750	+3236	− 43

Moreover, corresponding elastic constants for the two ends of the structure are nearly equal in each instance. But the experimental values, the averages for the two tests and for the two ends of the structure, given in Table 13, differ materially from the values determined by the elastic theory, which are given in italics. It is to be noted that for some of the elastic constants the computed values exceed the experimental values, for others the reverse is true.*

6. *Influence Ordinates for Reaction Components from Deflection of Structure.*—If the material in a structure follows Hooke's law, the influence ordinates for a reaction component at a terminal can be obtained by displacing the terminal in a direction parallel to the component being studied and measuring the accompanying vertical movement of the deck at each load point. This procedure was followed simultaneously with the tests to determine the elastic constants, described in Section 5. The resulting influence ordinates for the structure when free to sway are represented by the small circles of

*Tests and analyses described in Section 10 indicate that microscopic cracks, too small to be detected, affect the elastic behavior of a structure of this type, although they do not appreciably affect the moments due to loads at sections controlling the design.

TABLE 13

COMPARISON OF MEASURED AND COMPUTED ELASTIC CONSTANTS

Bases Fixed. Specimen 2.
Computed values are in italics.
Values for full width of structure, 18 in.

Movement of Base	Reaction Components; East Base		
	M ft. lb.	H lb.	V lb.
East Base Rotated $\Delta\theta = -0.001$ rad	−85 600 *−104 800*	4200 *5460*	980 *775*
West Base Rotated $\Delta y = -0.001$ rad	−38 700 *−67 500*	4200 *5460*	980 *775*
Settlement $\Delta y = 0.10$ in	8 000 *6 450*	20 *0*	340 *269*
Spread $\Delta x = 0.10$ in	−38 400 *−45 500*	3020 *3315*	36 *0*

Figs. 5 and 6. Diagrams are shown for Specimen 1 with bases hinged and also with bases fixed; Specimen 2 was tested with bases fixed only.

7. *Influence Ordinates for Reaction Components by Unit Loads.*—The influence ordinates for reaction components were determined by applying and removing a unit load of 2000 lb. at the various load points, successively, and weighing the accompanying changes in the reaction components. Two tests were made on Specimen 1 with the bases of the structure fixed and two with them hinged. The second test with the bases fixed was a check test; but, for the structure with hinged bases, the first test was made when the concrete was intact and the second test after the deck had been cracked by the application of one live load, as described in Section 15. The object of the latter test was to determine the effect of cracks in the deck upon the reactions due to loads.

The method of making a test was as follows. With the bases in their normal positions relative to each other a complete set of readings was recorded. This included all scales, Ames dials indicating the span, level bubbles indicating the angular positions of the bases, hook gages indicating the elevations of the bases relative to each other, and the Ames dials indicating the sway of the specimen. The 2000-lb. load was then applied at a load point and the resulting deformation of the apparatus supporting the bases corrected for by the following adjustments: The links connecting a base to its horizontal-load scales were adjusted for length until the Ames dials indicated that the

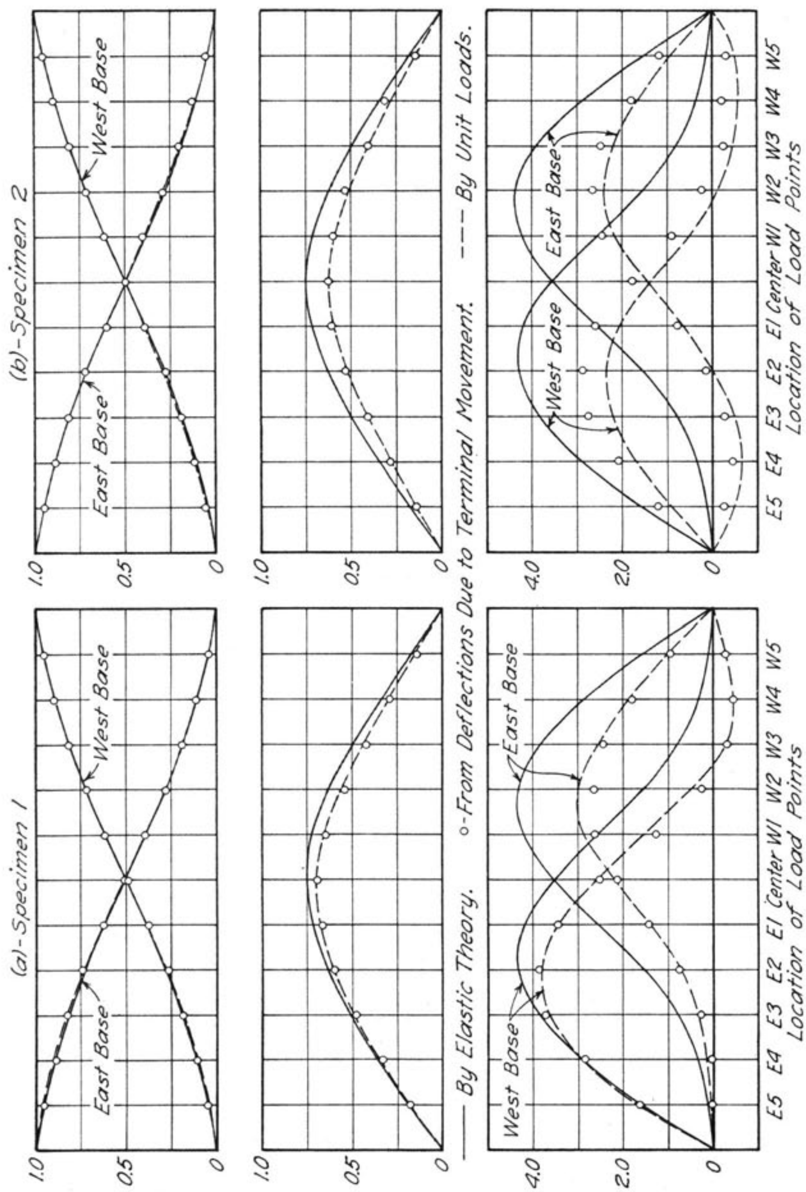


FIG. 5. INFLUENCE LINES BY VARIOUS METHODS; BASES FIXED, STRUCTURE FREE TO SWAY; SPECIMENS 1 AND 2

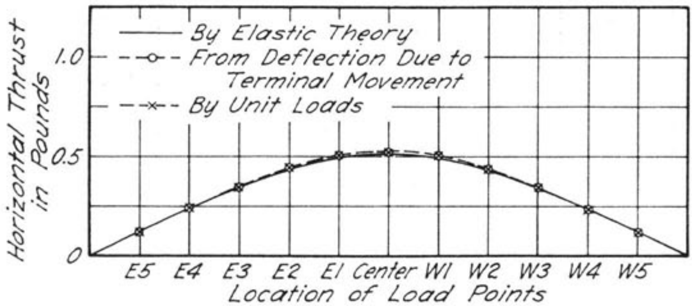


FIG. 6. INFLUENCE LINES BY VARIOUS METHODS; BASES HINGED, STRUCTURE FREE TO SWAY; SPECIMEN 1

length of the span had been brought to its original value; the jacks supporting the bases were manipulated until the hook gages indicated that the two bases had their original elevations relative to each other and, for the tests with fixed bases, the bubbles indicated that both bases had been returned to their original angular positions. When these adjustments had been completed another complete set of readings was recorded. The load was then removed, the bases returned to their original positions relative to each other, and a third set of readings was recorded. Thus for each of the load points, one test was made for determining the effect of applying the unit load and a second for determining the effect of removing the load. The results of a typical test, those for load point E3, are given in Table 14. Data similar to those contained in this table were obtained for all load points and the results are plotted as influence lines. Because the resulting influence lines for moments at the two bases differed considerably, the test was repeated for all load points. The influence lines determined by the two tests are compared in Fig. 7. Although the two tests do not check exactly, the two sets of influence lines do have the same general form.

The heavy dotted lines of Fig. 7 represent the average for the two tests, modified somewhat to make a smooth curve. They are reproduced as the light broken lines of Fig. 5a. This figure is for Specimen 1 with fixed bases, and shows influence lines for reaction components determined by various methods. Similar influence lines for Specimen 2 are given in Fig. 5b. The fact that, for Specimen 1, the two sets of influence lines, one determined by unit loads and the other from the deflections due to terminal moments, have the same general shape but both differ for the two ends, is additional evidence that, for this structure, the influence lines for the moments at the two bases do

TABLE 14
INFLUENCE ORDINATES BY UNIT LOADS
Load Point E3
Bases Fixed. Specimen Free to Sway
Specimen 1

Load	Change in Horizontal Thrust, lb.			Change in Vertical Reaction, lb.						Change in Moment, ft. lb.	
				East Base			West Base				
	East Base	West Base	Average	Outside Scale	Inside Scale	Total	Outside Scale	Inside Scale	Total	East Base	West Base
First Test											
On	+1020	+1030	+1030	+920	+730	+1650	+840	−480	+360	+1090	+7900
Off	−1010	−1030	−1020	−900	−740	−1640	−810	+450	−360	− 960	−7550
Average			+1020			+1650			+360	+1030	+7730
Second Test											
On	+ 920	+ 930	+ 930	+820	+820	+1640	+710	−360	+360	0	+6420
Off	− 990	−1000	−1000	−900	−750	−1650	−810	+450	−360	− 900	−7550
Average			+ 960			+1640			+360	+ 450	+6980
Average of two tests			+ 990			+1645			+360	+ 740	+7350

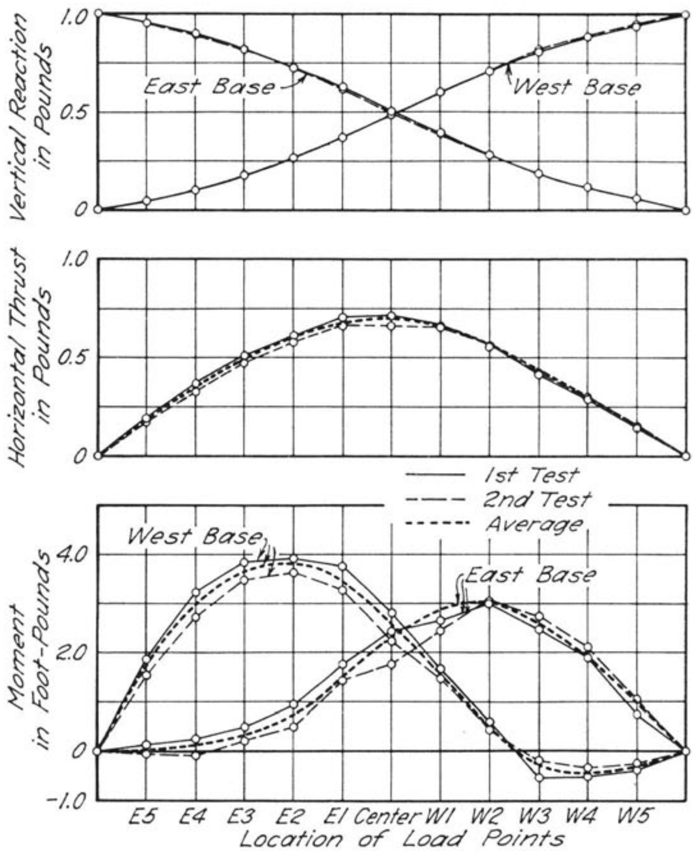


FIG. 7. INFLUENCE LINES BY UNIT LOADS; BASES FIXED, STRUCTURE FREE TO SWAY; SPECIMEN 1

really differ materially. For Specimen 2 for which the elastic constants for the two ends (Table 12) are nearly equal, the influence lines for the two ends, given in Fig. 5b, are more nearly of the same form, but the experimentally-determined sets of diagrams, one determined by the unit-load method and the other from deflections due to terminal movements, differ somewhat.

Two tests similar to the ones just described were made on Specimen 1 when the bases were hinged, one when the deck was intact, and the other after the concrete had been cracked by the application of the design live load. The influence lines representing the two tests, compared in Fig. 8, are nearly identical, indicating that the cracks in the

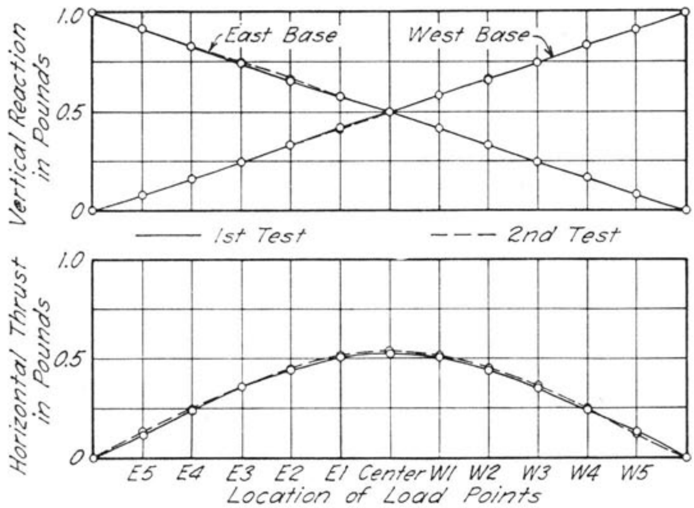


FIG. 8. INFLUENCE LINES BY UNIT LOADS; CRACKED AND UNCRACKED DECK; BASES HINGED

deck did not materially affect the influence lines for reaction components for the structure with bases hinged. The results, the averages for the two tests, are also represented by the light broken line of Fig. 6, in order that they may be compared with the corresponding values determined by other methods. The influence lines for the vertical reactions are not given because the lines determined by the various methods are so nearly identical that they are not distinguishable when plotted to the same axes.

A third test was made upon Specimen 1 with hinged bases to determine the reaction components at the bases when the structure was prevented from swaying by applying a longitudinal force at the west end of the deck 14.5 ft. above the base. The results of the tests are shown in Fig. 9.

The sway of the structure was measured during the test for determining the reaction components due to the application and removal of a unit load, both when the bases were fixed and when they were hinged. The resulting influence lines for sway are the broken lines of Fig. 10. The influence ordinates corresponding to the smooth curves are given in Table 15.

Knowing the influence ordinates for sway and the relation between the sway and the reaction components at the bases given in Table 11,

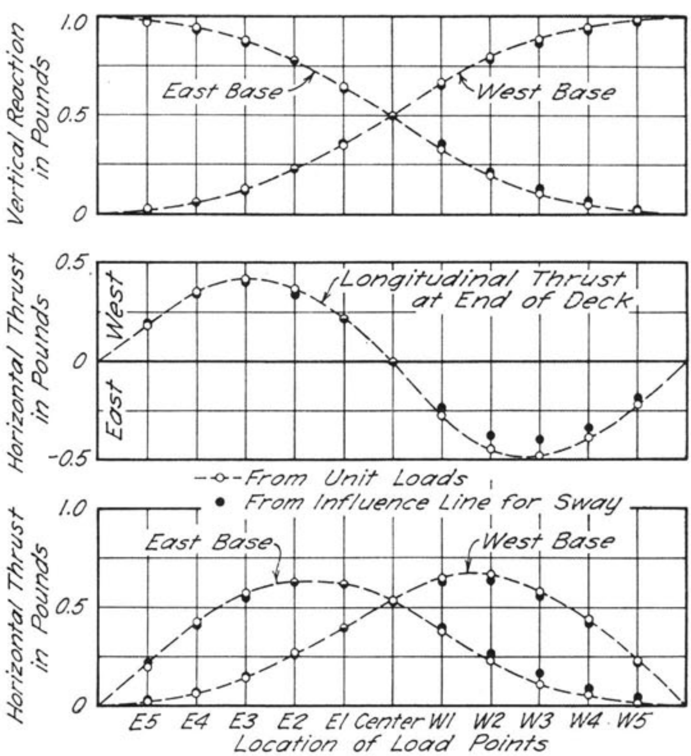


FIG. 9. INFLUENCE LINES BY UNIT LOADS; BASES HINGED, SWAY PREVENTED; SPECIMEN 1

the influence ordinates for the reaction components when sway is prevented can be determined from the corresponding quantities for the structure when free to sway. The small solid circles of Fig. 9 represent influence ordinates obtained by this method.

Tests were made to determine the changes in the reaction components accompanying the application and removal of a unit load for the structure with fixed bases and with sway prevented, but the results were so erratic that they have not been included in this report. The influence ordinates for this structure have, however, been computed from the following data: the influence ordinates when the structure is free to sway, given in Fig. 5, the relation between sway and the reaction components, given in Table 8, and the influence ordinates for sway given in Table 15. The resulting influence lines are given in Fig. 11, the left-hand diagrams being for Specimen 1, and the right-hand for Specimen 2.

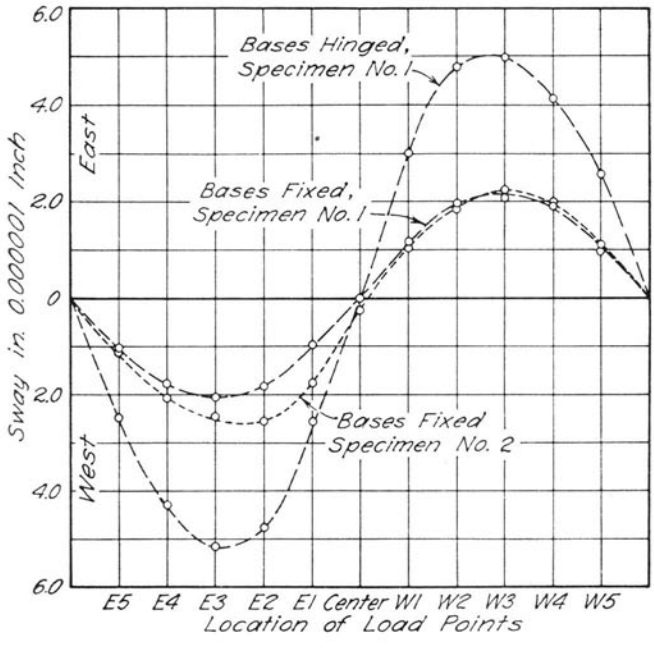


FIG. 10. INFLUENCE LINES FOR SWAY

TABLE 15
INFLUENCE ORDINATES FOR SWAY
A plus (+) sign indicates that the deck moved west.

1-lb. Load at	Horizontal Movement, 0.000 001 in.		
	Bases Fixed		Bases Hinged
	Spec. 1	Spec. 2	Spec. 1
E5	+1.08	+1.10	+2.44
E4	+1.74	+2.00	+4.30
E3	+2.07	+2.50	+5.12
E2	+1.76	+2.55	+4.72
E1	+0.94	+1.78	+2.80
Center	+0.00	+0.25	0.00
W1	−1.16	−1.08	−2.96
W2	−1.94	−1.85	−4.76
W3	−2.12	−2.23	−4.98
W4	−1.82	−1.97	−4.16
W5	−1.08	−1.10	−2.36

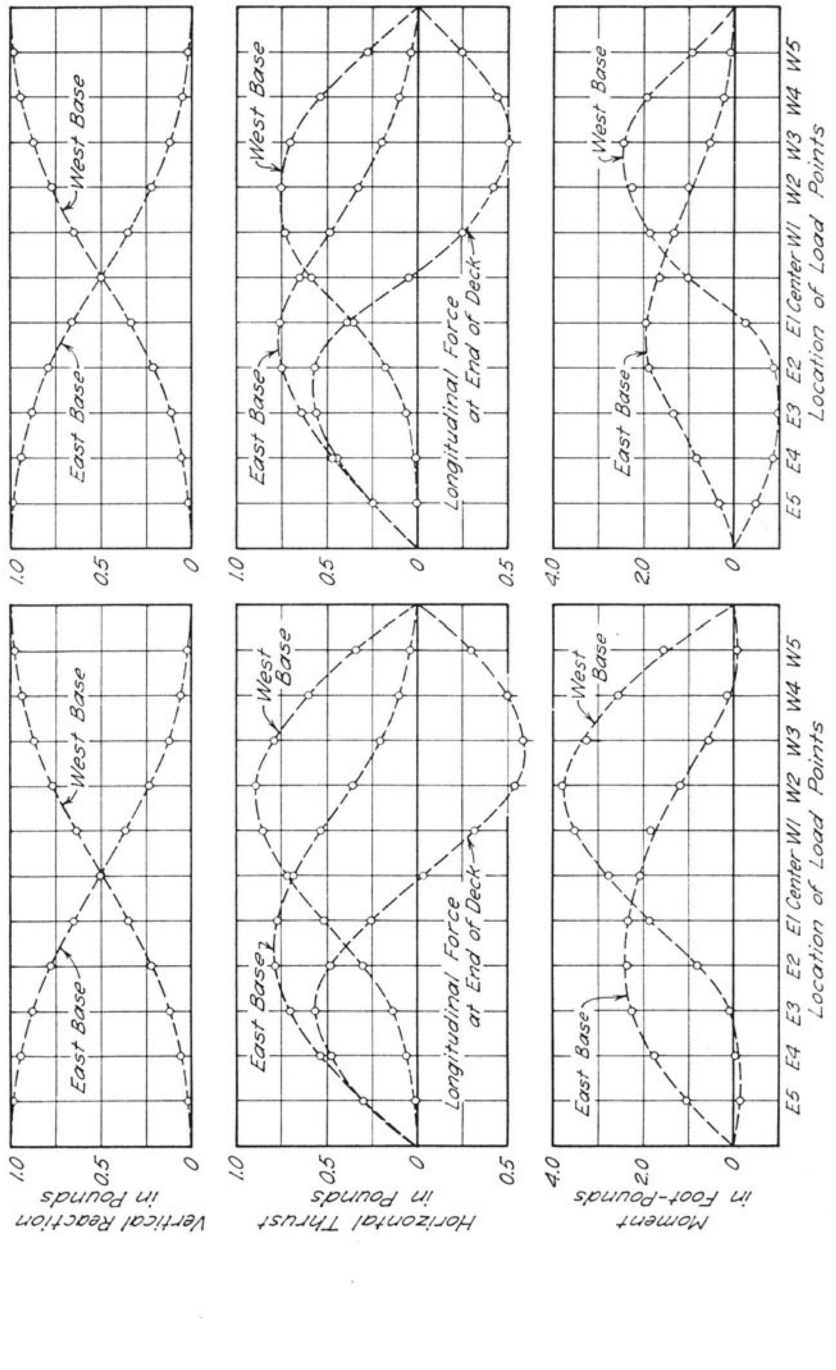


FIG. 11. INFLUENCE LINES FOR REACTION COMPONENTS; BASES FIXED, SWAY PREVENTED

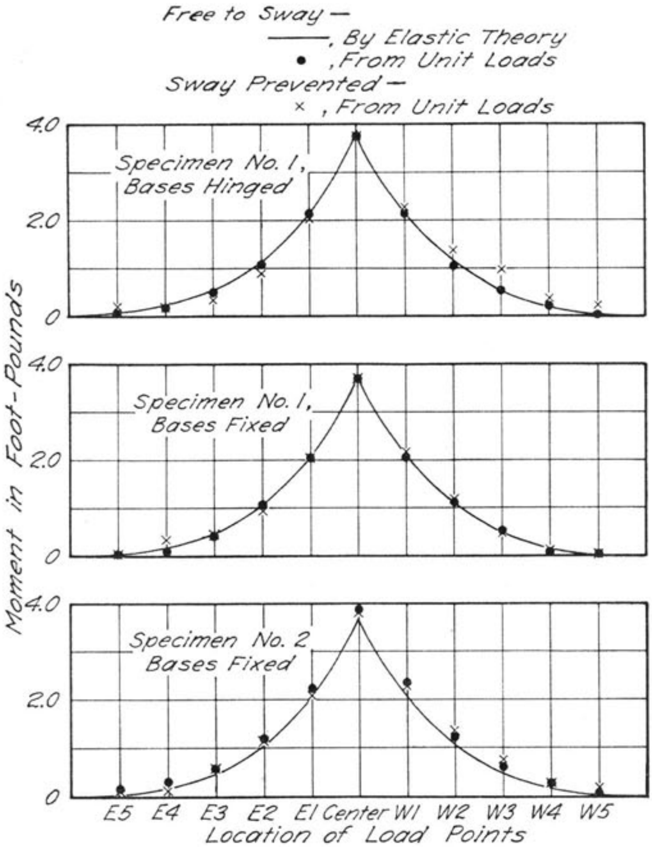


FIG. 12. INFLUENCE LINES FOR MOMENT AT CROWN

8. *Moments at Crown and Knee.*—The moments in the structure at the crown and knee were computed from the reaction components at the bases. These computations have been made using two sets of values, one determined by the elastic theory and given in Table 2, the other determined experimentally by the unit-load method and given in Fig. 5.

The influence lines for the moment at the crown are given in Fig. 12, for Specimen 1 and for Specimen 2. The full lines represent values computed by the elastic theory for a structure free to sway. The small solid circles represent values for the same structure determined experimentally. The two sets of values agree closely. The small crosses

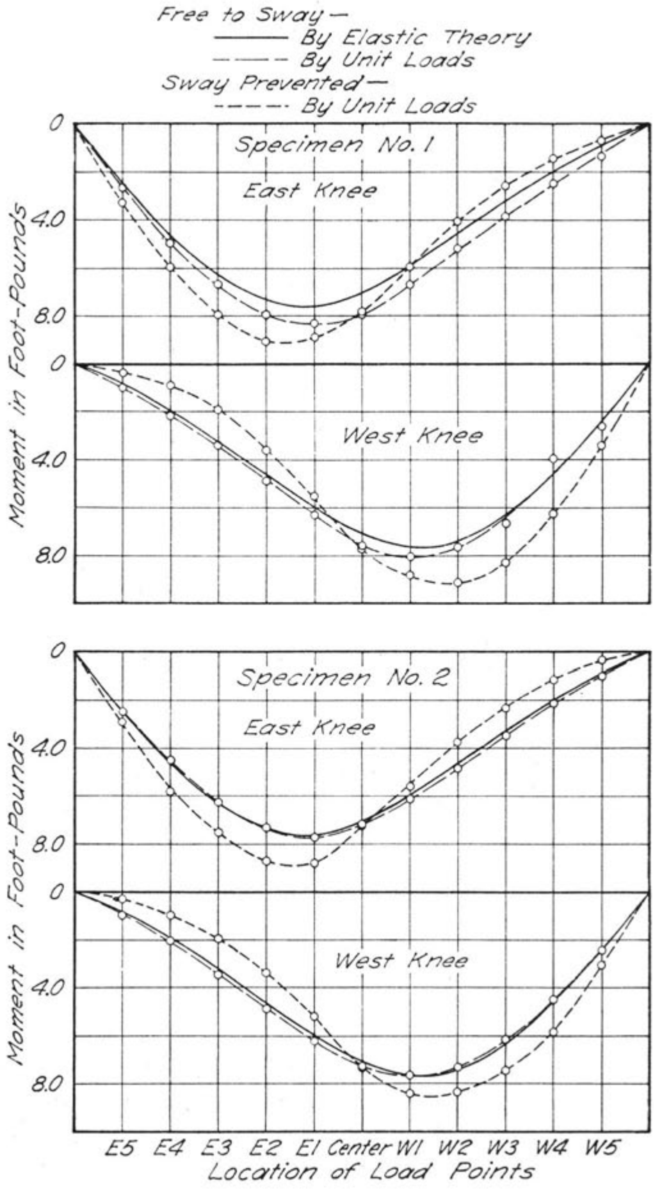


FIG. 13. INFLUENCE LINES FOR MOMENT AT KNEES; BASES FIXED

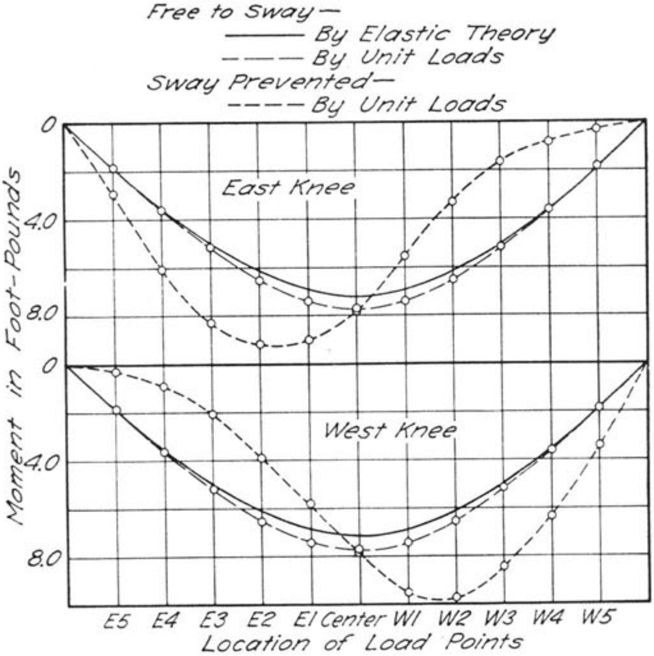


FIG. 14. INFLUENCE LINES FOR MOMENT AT KNEES; BASES HINGED

represent values determined experimentally for the structure restrained against sway. It would appear from these diagrams that, for this structure, preventing sway has very little effect upon the live-load moment at the crown.

The influence lines for the moment at the knees, for the structure with fixed bases, are given in Fig. 13. The full lines and the light broken lines are for the structure free to sway, the first being determined by the elastic theory and the latter by unit loads. The measured values are a little greater than the values determined by the elastic theory, but the difference is not great. The influence lines for the structure restrained against sway differ from the ones for the structure free to sway, but, for a symmetrical structure, only the live-load moment is affected as there is no dead-load sway. The live-load moment, however, is somewhat greater for the restrained structure than for the structure free to sway.

The influence lines for the moment at the knees, for Specimen 1 with hinged bases, are given in Fig. 14. For this structure, also, the measured values are somewhat greater than the values computed by the elastic theory and the influence lines are materially different for the restrained structure and for the structure free to sway. But, as for the structure with fixed bases, only the live-load moment is influenced by the restraint against sway as the dead load does not produce sway in a symmetrical structure. The maximum live-load moment may, however, be appreciably greater for the restrained structure than for the one free to sway.

The importance of the restraint against sway in the design and construction of a rigid frame bridge, depends upon the magnitude of the movement that would take place if the structure were free to sway. For if, with the structure free to sway, the movement is very small, it is highly probable that a considerable portion of the sway will occur even though there is nominal restraint. In the tests to determine the influence ordinates for reaction components by the application and removal of a unit load, described in Section 7, the magnitude of the sway was measured. The resulting influence lines for sway are given in Fig. 10. According to the experimentally-determined influence lines for Specimen 1, given in Figs. 13 and 14, the design live load when placed with the 6900-lb. load at W2 and the 1720-lb. load half way between E1 and E2, the positions of the loads for a maximum moment at the west knee for the structure with sway prevented, causes a movement of 0.011 in. for the structure with fixed bases and of 0.027 in. for the one with hinged bases. Any feature of the construction which will permit a sway of 0.01 or 0.02 in. will therefore reduce the moment at the knee. Nevertheless, the possibility that this restraint may cause a considerable increase in the moment at the knee should not be overlooked.

9. *Rotation of Hinged Bases.*—If the bases of a rigid frame bridge are hinged, they will rotate slightly due to loads, shrinkage, and temperature changes. In considering whether a structure functions more nearly as one with hinged or as one with fixed bases, it is desirable to know the magnitude of the rotation that would take place if the bases were hinged. In the tests of the structure with hinged bases to determine the reaction components by the application and removal of a unit load of 2000 lb., described in Section 7, the graduated bubble at each base was read to determine the magnitude of the rotation due to the load. The resulting influence lines for the rotation are given in Fig. 15. Spreading the bases of Specimen 1 0.10 in. produced a rotation of 0.00029 radian at each end of the structure. The maximum rotation to be expected due to dead and live load, temperature changes

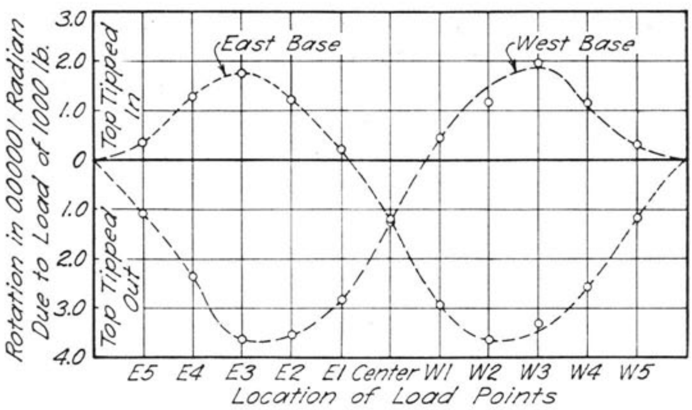


FIG. 15. INFLUENCE LINES FOR ROTATION OF BASES

and shrinkage, has been computed from the foregoing data for Specimen 1. For dead load the rotation is +0.00015 radian, and the rotation due to the design live load passing across the span varies from +0.0010 to —0.00025 radian. A temperature variation of ±45 deg. F. produces a rotation of ±0.00044 radian, and a shrinkage of 0.0003 in. per in. produces a rotation of +0.00048 radian. The maximum rotation due to these combined causes varies between +0.00207 and —0.00054 radian.

10. *Factors Affecting Moment in Rigid Frame Bridge with Fixed Bases.*—The influence lines of Fig. 5 indicate that the two sets of values for the reaction components at the bases of a rigid frame, one determined experimentally by the unit-load method and the other computed by the elastic theory, differed greatly for the frames tested, the experimentally-determined values being much less than the corresponding computed values. But the influence lines of Figs. 12, 13, and 14 indicate that the corresponding two sets of influence lines for the moments at the knee and at the crown are in close agreement. Moreover, there is nothing inconsistent in the two findings, since the small moment at the base is accompanied by a small horizontal thrust.

For a rigid frame bridge of the type tested a flexural failure at the knee would cause the structure to collapse. A flexural failure at the crown might injure the roadway, and would increase the moment at the knee somewhat, but would not, if the section at the crown retained its ability to resist shear and thrust, cause the structure to collapse. A flexure failure at the base would have but little effect upon the moment

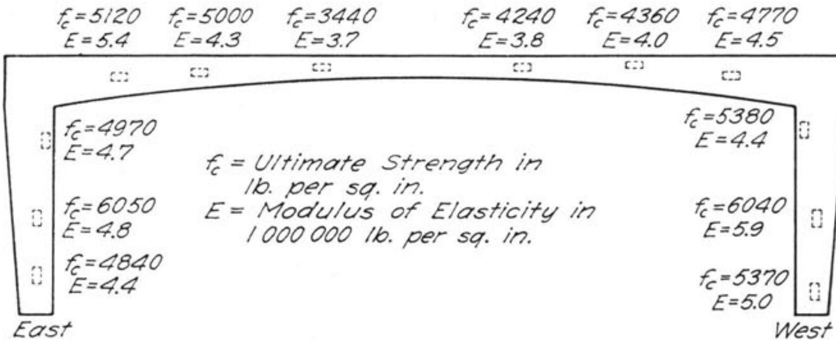


FIG. 16. VARIATIONS IN PHYSICAL PROPERTIES OF CONCRETE AS DETERMINED BY TEST CORES; SPECIMEN 1

at either the crown or the knee, providing the base retained its resistance to shear and thrust. For these reasons, although a fairly accurate determination of the moment at the crown and knees is desirable, the discrepancies between the measured and the computed values of the moments at the bases are of interest primarily as an academic question rather than as a question that vitally affects design. Nevertheless, extensive studies were made to determine why, for the structures tested, the measured and the computed values of the moments at the bases differed so greatly. These studies are described briefly in the following paragraphs.

A lack of homogeneity of the concrete might cause the moment at the bases due to a load on the deck to be different for the actual specimen from the moment for the hypothetical specimen of homogeneous concrete. For this reason, after the tests of Specimen 1 had been completed, 4-in. x 8-in. cores were cut from the deck and legs at various points, and tests were made to determine the strength and modulus of elasticity of the concrete of these cores. The results of these tests are presented in Fig. 16. It is to be noted that there is some variation in the individual values of E, but the variation is not much greater than is experienced in tests of cylinders made under laboratory conditions. The average value of E is about 10 per cent greater for the west than for the east leg, and the value for the central portion of the deck is only about 80 per cent of the value for the structure as a whole.

In order to determine the effect of variations in the modulus of elasticity of the concrete upon the influence ordinates for moments at the bases of the columns, analyses were made of several series of structures having the same dimensions as Specimens 1 and 2 but without reinforcement.* One of these structures, a structure for which *E* was assumed to have the same value at all sections, and for which the *I* was based upon the assumptions that the concrete was uncracked, that the neutral axes of the legs and deck coincided with the gravity axes up to the intersection of the latter, and that the two intersecting members maintained their separate identities up to the point of intersection, was designated as the "standard" structure. For the first series of structures, the values of *EI* were the same as the values of *EI* for corresponding sections of the standard structure, except for a portion 15.32 ft. long at the middle of the deck. For this latter portion, *R*, the ratio of the values of *EI* for the special structure to the values of *EI* for corresponding sections of the standard structure had, for the three structures of the series, values of 2.0, 1.0, and 0.7. For the second series of structures, the values of *EI* were the same as the values of *EI* for corresponding sections of the standard structure, except for a portion 7.66 ft. long at the bottom of each leg. For these portions, *R* for the three structures of the series had values of 2.0, 1.0, and 0.40. For the third series of structures, the values of *EI* were the same as the values of *EI* for corresponding sections of the standard structure, except for a block 4.21 ft. long at each end of the deck, three feet of which is also the top of the leg. For these blocks *R* for the three structures of the series had values of infinity, 1.00, and 0.25. All analyses are for structures with bases fixed and structure free to sway, no correction being made for rib shortening or shear deformation.

The influence lines for the moments at the bases of these special structures are given in Fig. 17. The upper group of lines shows the effect of variations in the values of *EI* over the central portion of the deck, the middle line of the group being for the standard structure. These diagrams indicate that decreasing the values of *EI* over the central portion of the deck increases the moments at the bases. It would appear, therefore, that the fact that the measured value of the moment at the base is less than the computed value could not be due to the low value of *E* over the central portion of the deck.

The middle group of curves in Fig. 17 shows the effect of variations in the values of *EI* over the lower portion of the legs, the middle curve of the group being for the standard structure. These curves indicate that increasing the values of *EI* over the lower portion of the legs increases the moment at the base due to load. The tests of the cores indicate that the modulus of elasticity of the concrete in the lower

*These analyses were made by A. A. Brielmaier, Associate in Civil Engineering, University of Illinois.

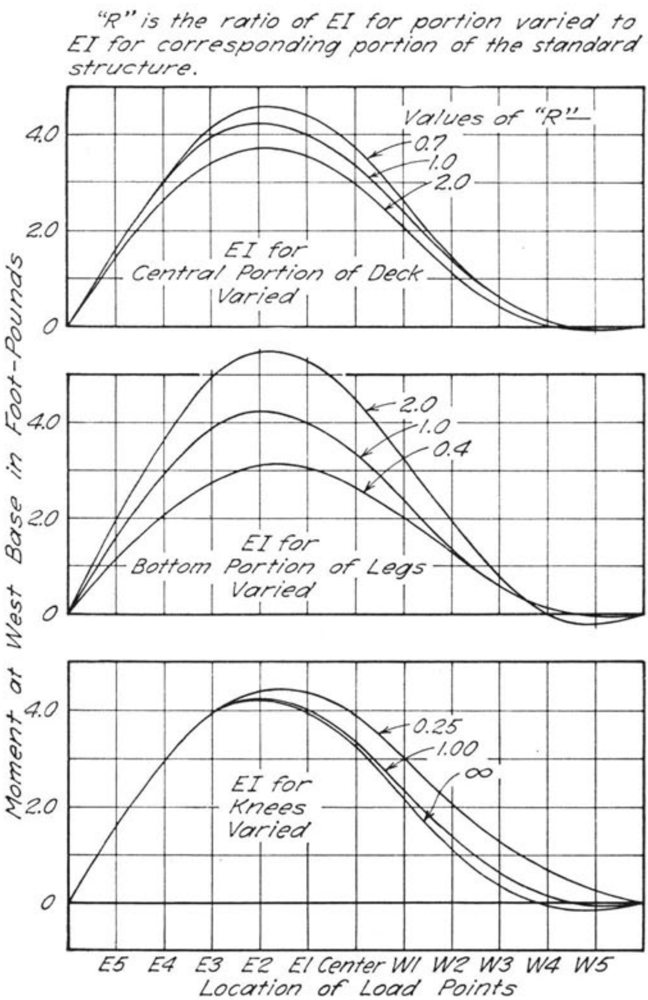


FIG. 17. EFFECT OF VARIATIONS IN EI UPON INFLUENCE ORDINATES FOR MOMENT AT BASES; BASES FIXED, STRUCTURE FREE TO SWAY

end of the legs of Specimen 1 was actually greater instead of less than the average for the whole structure, so that the discrepancy between the computed and the measured values of the moment at the base could not be due to variations in the value of E for the concrete near the bases.

The lower group of curves in Fig. 17 shows the effect of variations in the values of EI at the knees upon the influence lines for moments at the bases. As in the other groups, the middle curve is for the standard structure. Because of the uncertain stress condition at a knee, the range of EI for the portion varied was greater than for the other groups of curves. With R equal to infinity the vertical section of the deck inside of the column and the horizontal section of the column at the underside of the deck have the same angular motion, the stiffest possible condition. Stiffening the knees decreases the moments at the bases but changing the knees from the standard to the stiffest possible condition reduces the moments by only a very small part of the difference between the measured and the computed values for Specimens 1 and 2.

As a result of the foregoing study it would appear that the discrepancy between the measured and the computed values of the influence ordinates for moments at the bases is not due to variations in the E of the concrete.

The analyses that have been described in the previous paragraphs indicate that the smallness of the moments at the bases was not due to a low modulus for the concrete near the bases. But the middle group of curves in Fig. 17 suggests that it might be due to a small value of the effective I near the bases. And the effective value of I for a reinforced concrete member can be reduced by either a bond failure or by cracking the concrete. Both of these factors would reduce the effective I of the section.

The reinforcing extends 20 inches below the theoretical base of the legs. Moreover, the reinforcing is anchored so that, as it is believed, a bond failure would be very improbable.

Specimens 1 and 2 were not knowingly subjected to loads or base displacements that would crack the concrete in the lower portion of the legs prior to the unit-load tests for determining the influence ordinates for moments at the bases. Moreover, the lower portions of the legs were carefully examined for cracks and none were detected. So there was no reason, other than the discrepancy between the measured and the computed values of the influence ordinates for moments at the bases, for believing that the concrete was cracked. But because of the unexplained discrepancy, another test was planned in which bond slip and undetected cracking of the specimen were, it is believed, definitely eliminated. The specimen was a $\frac{1}{8}$-size model of Specimens 1 and 2, and was designated as Specimen 3. It was made of a mix of sand and cement, had no reinforcement, and was cast in a flat position, the material being tamped systematically so as to obtain, as nearly as possible under laboratory conditions, a uniform E for the structure.

It was thought that, by omitting the reinforcement, not only the possibility of slip but also the possibility of undetected cracks would be eliminated, for, without reinforcement, a crack would result in fracture.

Tests of Specimen 3 gave values for the moments at the bases due to load on the deck that agreed well with values determined by the elastic theory. Another specimen was made similar to Specimen 3 except that it was reinforced. Tests of this specimen also gave values of the moments at the bases due to a load on the deck that agreed well with values determined by the elastic theory.

As a result of this extended consideration of the situation, the most plausible explanation of the discrepancy between the measured and the computed values for the moments at the bases of Specimens 1 and 2 would seem to be that the concrete was cracked in the lower portion of the legs, the cracks being so fine that they were not discernible. Specimen 2 was therefore subjected to a second unit-load test planned so as to disclose any discontinuity that might exist in the concrete. Sensitive bubbles were attached to the legs of the specimen, one at the bases where the rotation was prevented, and others at frequent intervals along the legs above the bases. This test established the fact that the rotation relative to the base of a horizontal section of the leg 6 in. above the base was several times greater than the rotation that could be accounted for by the elastic deformation of the concrete. With this additional information relative to its location a crack was finally detected in the west leg with the aid of a high-powered microscope. It was so small, however, that even after being located with the microscope it could hardly be discerned with the unaided eye.

The structures that were made the subject of the analytical study to determine the effect of variations in EI upon the moments at the bases were also used in similar studies relative to the moments at the knees and the crown. The results of these studies are presented in Fig. 18. The influence lines for moments at the knees, given in Fig. 18a, indicate that variations in the E of concrete that are likely to occur in the construction of structures of this type do not have an appreciable effect upon the moments at the knees. Likewise, the influence lines for moment at the crown, given in Fig. 18b, indicate that variations in the E of concrete that are likely to occur in the construction of structures of this type do not appreciably affect the moment at the crown.

As stated previously, the discrepancy between the measured and the computed values of the moments at the bases is of interest primarily as an academic question. However, because of the uncertainty

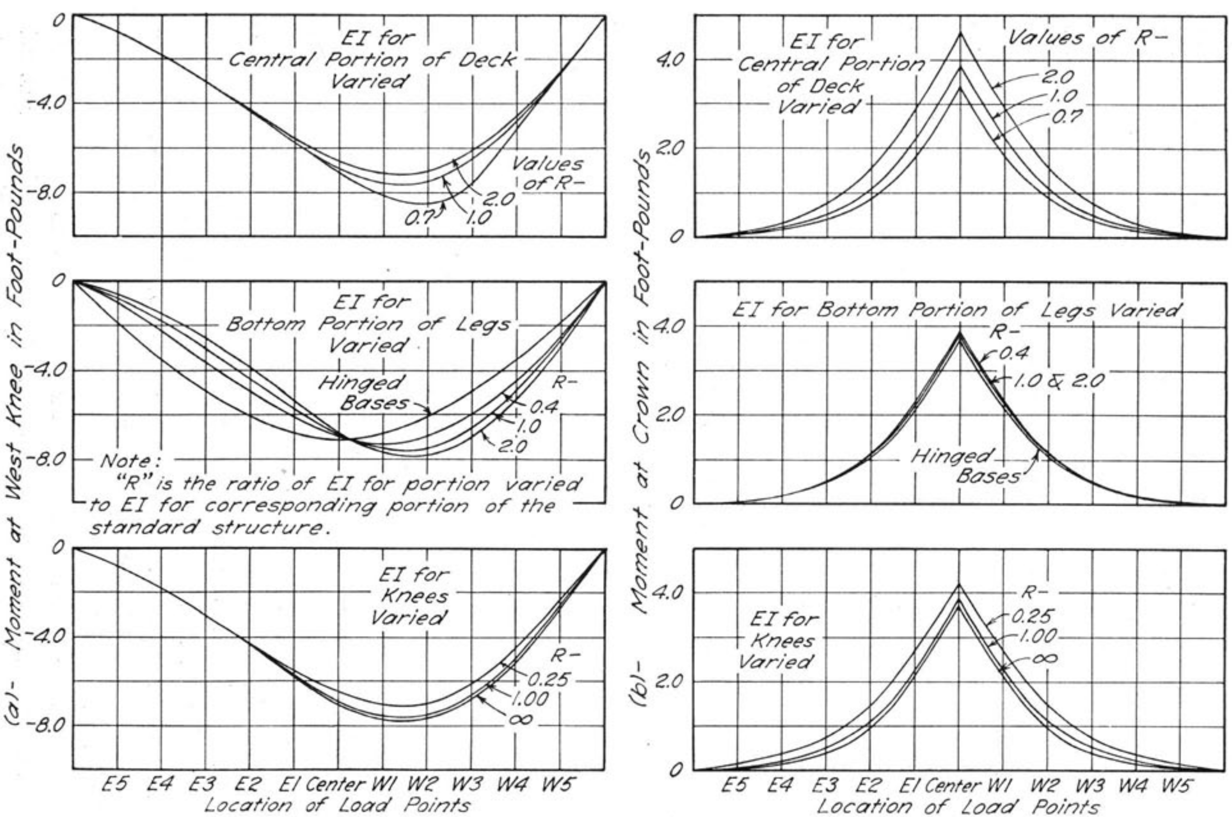


FIG. 18. EFFECT OF VARIATIONS IN EI UPON INFLUENCE ORDINATES FOR MOMENT AT KNEES AND FOR MOMENT AT CROWN; BASES FIXED, STRUCTURE FREE TO SWAY

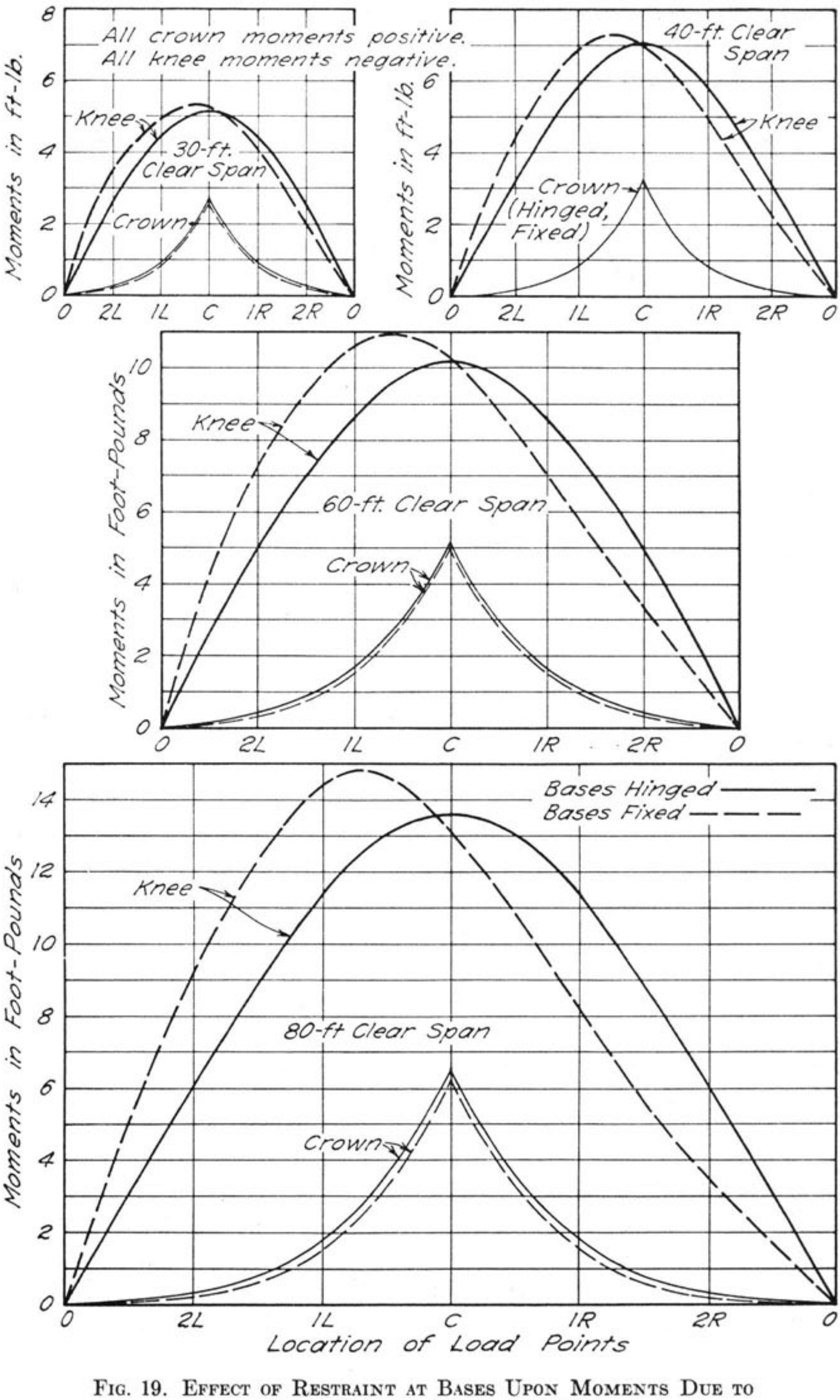


FIG. 19. EFFECT OF RESTRAINT AT BASES UPON MOMENTS DUE TO LOADS; VARIOUS VALUES OF H/L

that is sometimes expressed relative to the applicability of the elastic theory to the analysis of the rigid frame, it appeared desirable to study the question at some length. The results of this study have led to the following conclusions: (1) For an uncracked specimen the moment, thrust, and shear at any section due to loads on the deck agree with the values of the corresponding moment, thrust, and shear determined by the elastic theory. (2) Cracks near the base of a leg reduce the moment at the base, but do not cause an appreciable change in the load moment at other sections.

11. *Effect of Hinged Bases on Moment in Rigid Frame Bridge.*—Specimen 1 was tested when the bases were fixed and also when they were hinged.

The influence line for the moment at the crown is given in the upper part of Fig. 12 for the structure with hinged bases. The full lines represent values determined by the elastic theory and the small circles represent values determined experimentally by the unit-load method. The results obtained by the two methods are seen to be in almost perfect agreement. Moreover, the influence lines for the crown moment for the structures with fixed and with hinged bases are very nearly alike. This is in agreement with the data in Table 4, which indicate that the maximum resultant moment at the crown (dead-load, live-load, shrinkage, and temperature moment) is not materially affected by the angular restraint at the bases.

The influence lines for the moments at the knees are given in Fig. 14 for the structure with hinged bases. The light broken line represents experimentally-determined values, and the full lines represent values determined by the elastic theory. The two sets of values agree closely. The corresponding influence lines for the structure with fixed bases are given in Fig. 13. A comparison of the influence lines of Figs. 13 and 14 shows that the angular restraint at the bases had very little effect upon the moment at the knees. This statement also is consistent with the data in Table 4. In view of this statement, together with the last statement in the preceding paragraph, it appears that the angular restraint of the bases has very little effect upon the moment on a section at either the crown or the knee of the rigid frame bridge tested.

Because the effect of angular restraint at the bases upon the moments at the crown and knee might be affected by the relation of the height to the span, analyses were made of other rigid frames. All were 20 ft. high, and the spans varied from 30 ft. to 80 ft. Influence lines for the moments at the crown and the knee are compared in Fig. 19 for structures with fixed and with hinged bases. From an examination of these diagrams it appears that live-load moments at the knee and crown are very little affected by the angular restraint of the bases, and if dead load, temperature, and shrinkage moments are combined with the live load moment, the resultant design moments, likewise, are not affected much by restraint. Except for two instances the differences in moments are all less than 10 per cent. For moment at the knee for 60- and 80-ft. spans the differences in moments are 15 per cent and 13 per cent, respectively, which is not very large when one considers that they represent an extreme range from complete flexibility to complete restraint at the bases. Undoubtedly some rotation of footings intended to be fixed does occur. It would, however, appear that an unexpected angular movement at the bases of a rigid frame should be of little concern as far as stress at the critical points of the structure are concerned. Likewise, it would appear that hinges at the bases of the frame need not be completely flexible, provided the vertical leg is adequately reinforced, as angular restraint at the bases does not affect the moment appreciably except near the bases.

12. *Temperature and Shrinkage Stresses.*—If the deck of a rigid frame bridge changes in length due to any cause and the distance between the bases remains constant, flexural stresses are produced in the structure. The two most important influences that change the length of the deck are the shrinkage of the concrete and the expansion and contraction that accompany changes in temperature. The magnitude of the shrinkage varies with the character of the concrete and its exposure to drying influences. A shrinkage of 0.0003 in. per in. is as great as is likely to occur in the deck of a rigid frame bridge except under unusual conditions. Both the thermal coefficient for concrete and the temperature range to which the concrete is subjected are somewhat uncertain, but, for central Illinois, a temperature range of $\pm$ 45 deg. F. and a thermal coefficient of 0.000006 are acceptable. This corresponds to a unit variation from the mean of 0.00027 in. per in. length of span.

Table 4 presents the moments at the crown, knee, and base due to a shrinkage coefficient of 0.0003 and to a unit change due to temperature variations of $\pm$ 0.00027. The values given were determined by the elastic theory on the basis that E for the concrete was 3 000 000 lb. per sq. in. The temperature and shrinkage stresses at the bases are large for a structure with fixed bases, but, relative to the total stress, they are not large at the knee and crown. There are two influences that reduce the moments at the bases. One is a lack of rigidity in the foundation, and the other is a reduction of the EI of the lower portion of the legs due either to plastic flow or to the cracking of the concrete. The total rotation of a base due to dead load, live load, shrinkage, and temperature changes, given in Section 9, is + 0.0026 radian for the structure with hinged bases. This is equivalent to a rise of 0.19 in. on the outer edge and an equal settlement on the inner edge of a base 12 feet wide. Moreover, if the concrete at the base of the vertical leg flows because of the moment resulting from the angular restraint of the base, this flow automatically reduces the moment at the base without greatly changing the moment distribution at other portions of the structure. It would appear, therefore, that so long as the bases do not spread or settle, it is immaterial for the structure tested whether the bases are really fixed or hinged if the bottoms of the vertical legs are designed for fixed bases.

A test to determine the effect of time yield in the concrete on the temperature stresses is reported in Section 13.

13. *Effect of Time Yield in Concrete on Temperature Stresses.*—After the tests of Specimen 2, which are described in Sections 5 to 8, had been completed, a test was made to determine the effect of time yield in the concrete on temperature stresses. Instead of subjecting the specimen to a change in temperature at a constant span, the temperature was held constant and the span was changed. The test was planned to simulate in this manner the temperature changes occurring during three complete yearly cycles, the three cycles being completed in 462 days beginning when the specimen was 134 days old. The change in span corresponding to the maximum change in temperature was taken as 0.36 in., or 0.000625 in. per in. This corresponds to a change in temperature of 104 deg. F. if the thermal coefficient is taken as 0.0000060. This change in span was made in six increments of 0.06 in. each at intervals of approximately 10 days, individual time increments being varied somewhat so that readings could be taken when the temperature in the laboratory had a predetermined value.

The method of making the test was as follows: The bases of the specimen were fixed, and the specimen carried only its own weight. For the first set of readings the bases were in their normal positions relative to each other, and they retained these relative positions throughout the test except that the span was systematically changed as indicated in the following. After the reactions of the bases had been recorded, the span was increased 0.06 in., and the reactions were again recorded. The resulting changes in the reactions were the immediate effect of the change in span. The span was then allowed to remain constant for a period of 10 days and the reactions were again noted.

This change was due to the time yield in the concrete. The span was again increased 0.06 in. and the reactions recorded immediately after the change, and again at the end of a 10-day period. This process was repeated until the span had been increased to 0.18 in. more than normal. The process was then reversed and continued until, after 6 increments, the span was 0.18 in. less than normal. The test was continued until the equivalent of three yearly cycles had been produced. The results of the test are presented graphically in Fig. 20.

Figure 20a represents the horizontal thrust at the bases. The ordinate *a b* represents the thrust due to dead load, and *b c* the decrease in thrust due to the first increase in span of 0.06 in. The line *c d* indicates that the time yield in the concrete caused the thrust to increase even though neither the load nor the span (temperature) had changed. The subsequent variations in the thrust during the remainder of the test are shown by the diagram. The full lines represent the measured values of the thrust and the dotted lines represent the measured values modified by the change in thrust resulting from time yield.

It is of interest to note that during those portions of the cycle in which the span was being increased, thus reducing the thrust, the effect of time yield was to increase the thrust; and during those portions of the cycle in which the span was being decreased, thus increasing the thrust, the effect of time yield was to decrease the thrust. That is, the time yield in the concrete occurring between span increments of the same sign partially annuls the effect of the span increments. But the fact that the slope of the inclined lines decreases with repetitions of the annual cycle indicates that the effect of the time yield becomes somewhat less as the concrete ages. The difference between the amplitudes of the full-line and of the broken-line diagrams indicates the effect of time yield upon the annual variations in the temperature thrust.

Figure 20b shows the effect of time yield upon the moments at the bases due to temperature changes (changes in span). The ordinate *a b* represents the positive dead-load movement, *b c* represents the algebraic decrease in moment due to an increase in span of 0.06 in., and *c d* represents the algebraic increase in the moment during the 10-day period during which neither the load nor the span was changed. This change in moment is attributed to the time yield in concrete. The discussion of the thrust diagrams of Fig. 20a given in the previous paragraph applies equally well to the moment diagrams of Fig. 20b.

The variation in the temperature moments at the crown and knees

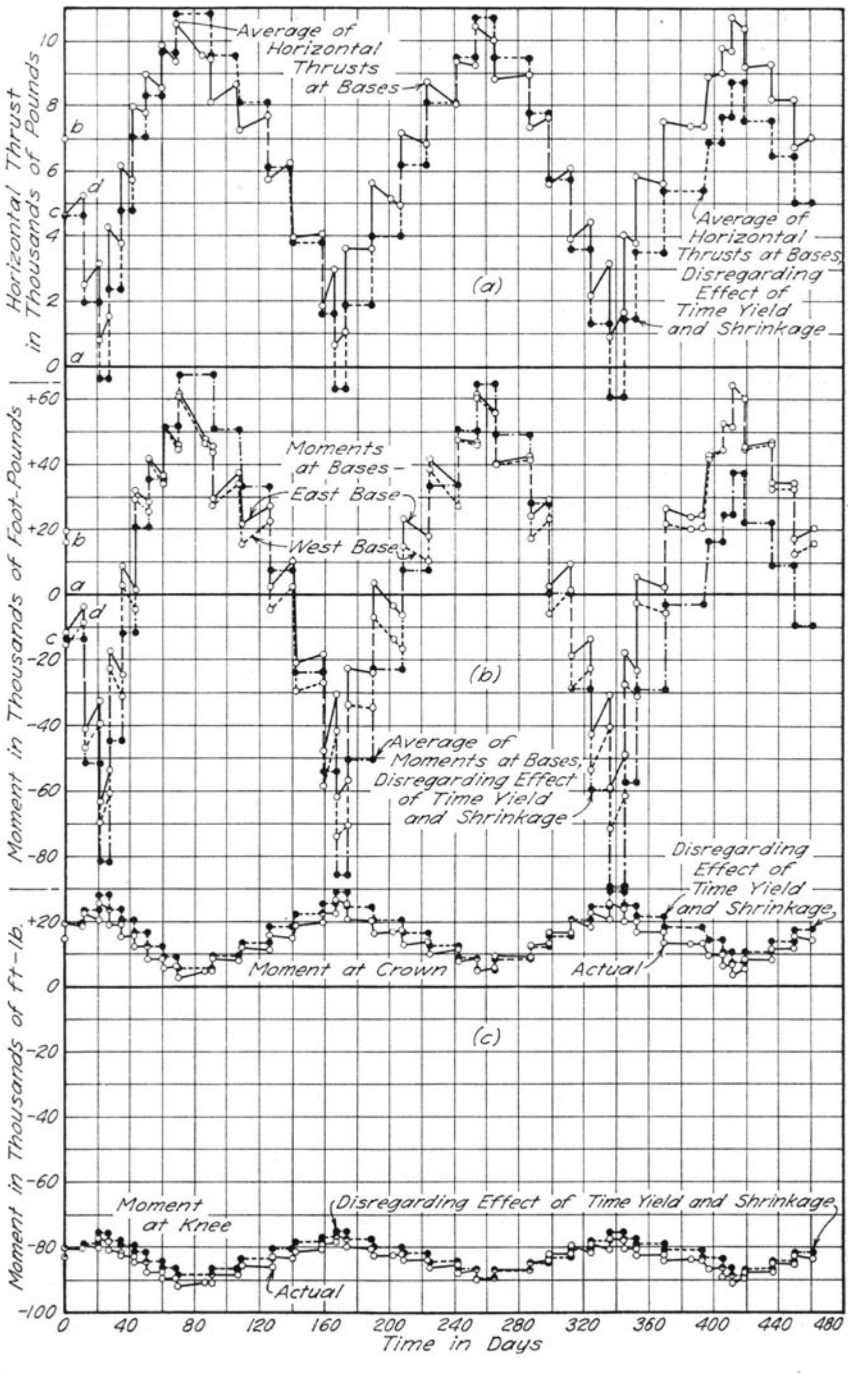


FIG. 20. EFFECT OF TIME YIELD IN CONCRETE UPON TEMPERATURE THRUST, TEMPERATURE MOMENT AT BASES, AND TEMPERATURE MOMENT AT CROWN AND KNEE; SPECIMEN 2; BASES FIXED

is shown in Fig. 20c. It is to be noted that these moments are smaller than the moments at the bases due to the fact that the elastic center is considerably above mid-height of the structure. It is also to be noted that the effect of time yield upon the moments was less at the crown and knees than at the bases.

As a result of these tests it is apparent that time yield in the concrete relieves the temperature stresses somewhat during the first two or three years after a reinforced concrete rigid frame is poured, but that this reducing effect gradually decreases as the structure becomes older.

14. *Effect of Gradual Spreading of Bases Upon Stresses in Rigid Frame Bridge.*—After the tests described in Section 13 had been completed, and when the structure was approximately 600 days old, Specimen 2 was subjected to a test to determine the effect of a gradual movement of the bases upon the reactions at the bases and upon the moments at the knees and crown. During the early part of the test the bases were fixed against rotation, later the test was continued with the bases hinged. At the beginning of the test the bases were brought to their normal position relative to each other, and the design live load, 6900 lb. at the middle of the deck, and 1720 lb. 14 ft. east of the middle, was placed on the structure, and the deflection of the deck and the reactions at the bases were recorded. The bases were then allowed to spread 0.10 in. without rotation or vertical movement, and the reactions at the bases and the deflection of the deck were recorded. The changes noted were the immediate effect of the spread of the bases. At the end of a week the readings were again recorded, the changes being attributed to time yield in the concrete. After these latter readings had been taken, the span was again increased 0.10 in., and the readings were recorded immediately after the change had taken place and again at the end of a week. The test was continued in this manner until the span had been increased 0.76 in., at which time the horizontal thrust was zero. Then, with all horizontal restraint removed, but with the bases fixed against rotation, the specimen carrying the design live load was allowed to stand for a period of 42 days. During that time the span increased 0.061 in., the spread being only 0.007 in. during the last 14 days, and 0.003 in. during the last 7 days.

Since no further appreciable spread could be expected with the bases fixed, the test was changed into one in which the bases were hinged. The first readings for the new test were taken when the specimen carried no load except its own weight, and when the span was

0.80 in. more than normal. After the readings under these conditions had been recorded, the design live load was applied and the readings were recorded, first with the span 0.80 in. more than normal, and again immediately after the spread had been increased to 1.00 in. The readings were again recorded after the specimen had been subjected to a spread of 1.00 in. for 7 days, and then immediately after the spread had been increased to 1.25 in. The test was continued in this way, increasing the spread in increments of 0.25 in. each week, and reading the reactions and deflection immediately after and 7 days after each increment in span had been allowed to take place. This procedure was continued until the total spread had increased to 4 in.

The effect of the gradual increase in span upon the reactions at the bases and upon the moments at the crown and knees is shown in Fig. 21, and the cracks resulting from the increase in span are shown in Fig. 22.

The portion A B of the lower diagram of Fig. 21a shows the reduction in the thrust as the span was allowed to increase with the bases fixed against rotation until the thrust was finally zero. The vertical portions of this diagram indicate the immediate reduction in thrust with an increase in span of 0.10 in., and the portions of the lines inclined upward and to the right indicate the recovery in the thrust during the 7-day periods when there was no change in the span. The vertical portion C D of the diagram indicates the increase in the dead-load thrust due to changing the structure from one with fixed to one with hinged bases, and the portion D E indicates the increase in the thrust due to the application of the live load. The line E F indicates the gradual falling off of the thrust as the span was increased 0.25 in. per week. The upper diagram of Fig. 21a shows the corresponding changes in the moment at the bases.

The variations in the moments at the crown and knees are represented graphically by the diagrams of Fig. 21b, the upper diagram being for the moment at the crown and the lower one for the moment at the knees. In these diagrams the vertical lines represent the immediate effect of the changes in span, and the inclined lines represent the recovery between successive increments in span. It is of interest to note that, in general, the recovery is from one-quarter to one-third of the immediate effect of the change in span, indicating that the time yield appreciably reduces the stresses resulting from a gradual spreading of the bases.

The quite uniform distribution of the cracks in the deck resulting from the spread of the bases, shown in Fig. 22, is of interest. The

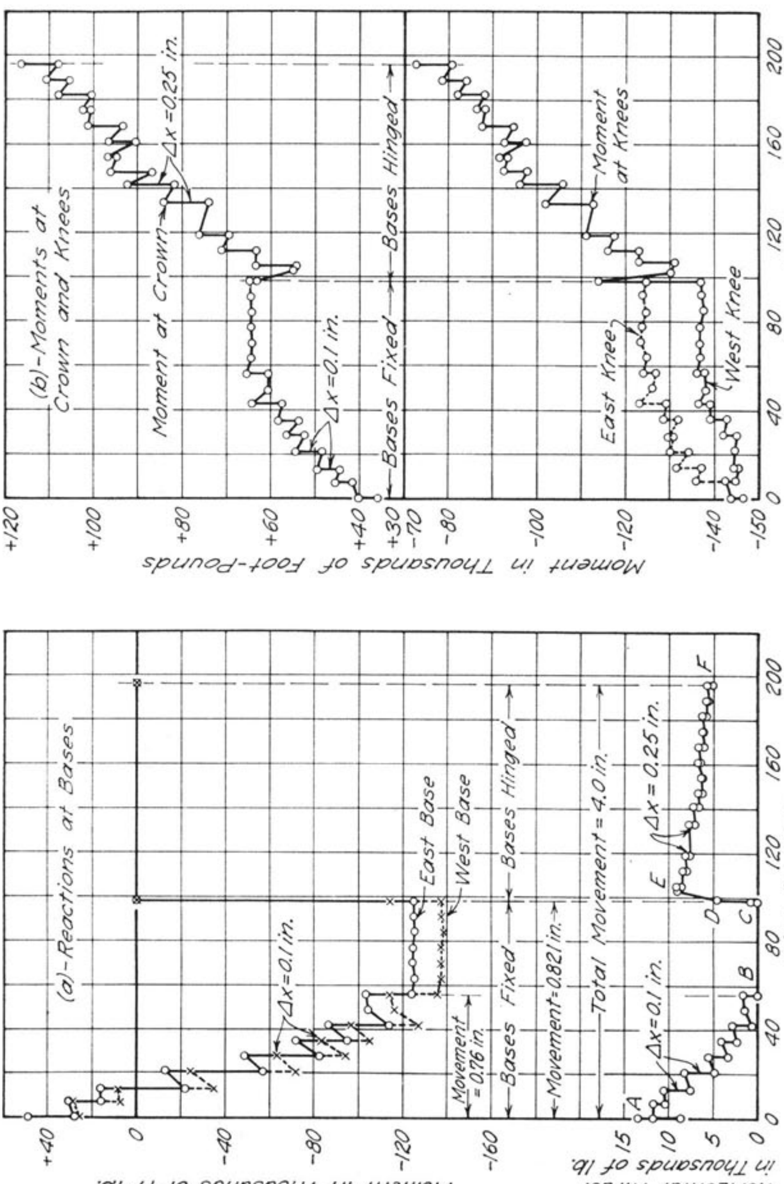


FIG. 21. EFFECT OF GRADUAL CHANGE IN SPAN UPON REACTIONS AT BASES, AND UPON MOMENT AT CROWN AND KNEE; SPECIMEN 2

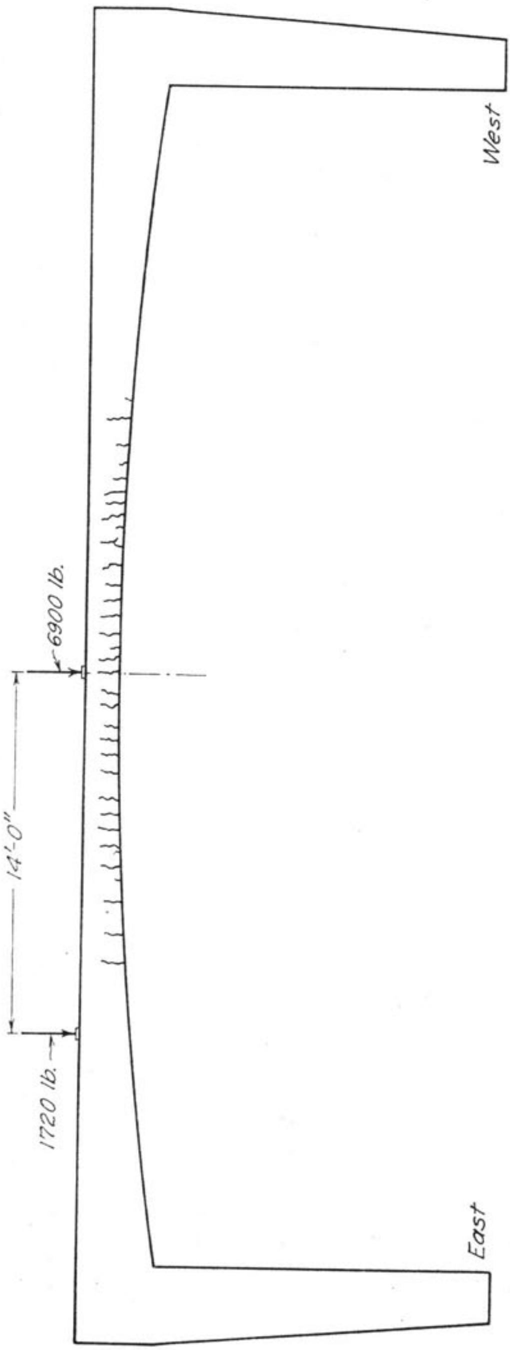


Fig. 22. Cracks Due to Gradual Spreading of Bases

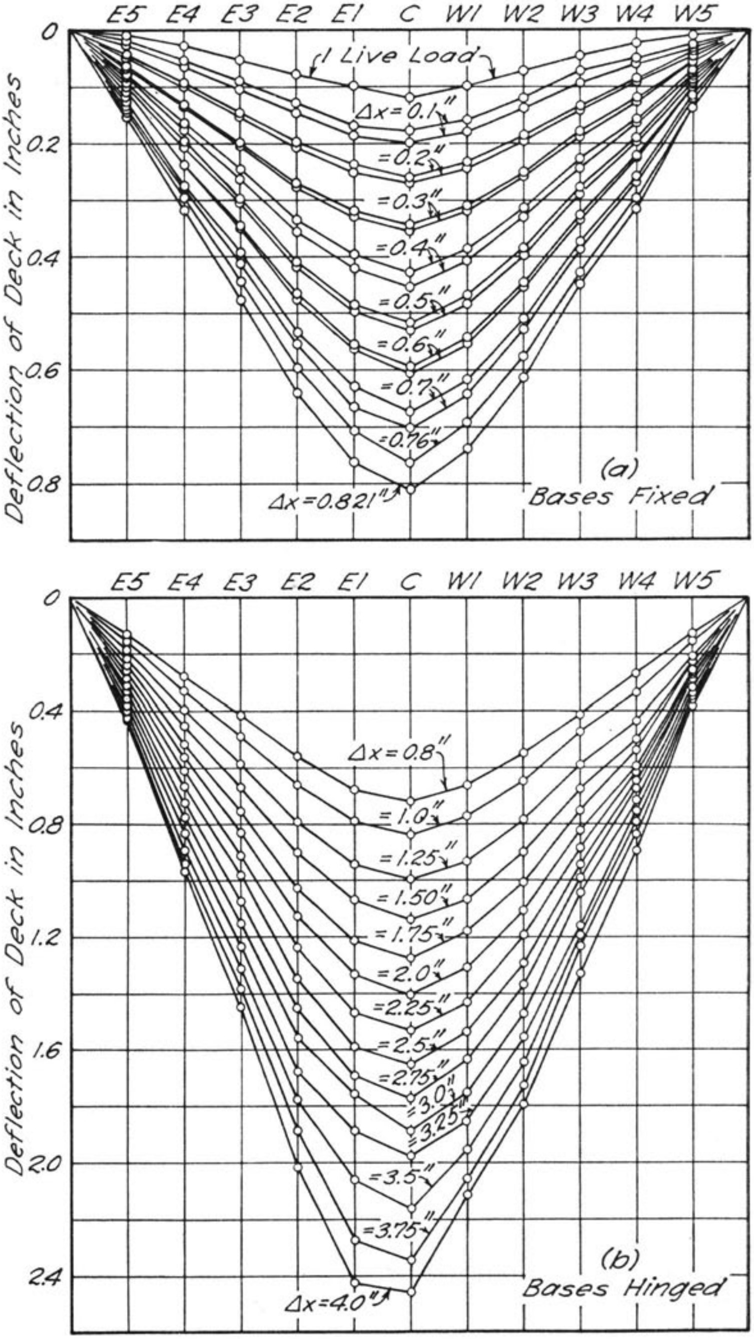


FIG. 23. DEFLECTION OF DECK DUE TO GRADUAL SPREADING OF BASES, WITH BASES FIXED AND WITH BASES HINGED

cracks were quite small and quite uniform, the largest directly under the large concentrated live load at the center being only 0.008 in. wide. The moment at the crown with the bases spread 4 in., given in the upper diagram of Fig. 21b, was slightly less than 117 000 ft. lb., a value not great enough to produce a yield-point stress in the steel.

The deflection of the deck due to the gradual spreading of the bases is shown in Fig. 23, for the structure with fixed bases and for the structure with hinged bases. There are two deflection diagrams for each increment of spread for the early part of the test. One represents the deflection immediately after the spread has been allowed to take place, and the other at the end of a 7-day period. The difference between the two represents the deflection due to time yield. The deflection due to time yield, with the bases hinged, was so small that it has not been indicated in the figure.

15. *Design-Load Test.*—Specimen 1 was tested to determine the reaction components at the bases due to the live load for which the structure was designed. This load represents the axle loads, including impact, of a standard 20-ton truck distributed across a 9-ft. width of roadway and, for the 1.5-ft. width of the specimen, consisted of two loads, 1720 lb. and 6900 lb., respectively, spaced 14 ft. apart. The structure was tested first with hinged and then with fixed bases, and was free to sway in both instances. The loads were placed with the larger load at the center and the smaller one 14 ft. east of the center. This position produces a maximum moment at the crown for both structures and also produces a moment only slightly less than the maximum moment at the knee for the structure with hinged bases. The method of loading is illustrated in Fig. 4 and described in Section 4.

The testing procedure was similar to that followed in determining the reaction components due to unit load, described in Section 7, with the addition that strains were measured with an 8-in. strain gage. The bases were returned to their normal positions relative to each other after each load change before the instruments were read. Two tests were made, one to determine the changes that accompanied the application of the design live load, and the other to determine the changes that accompanied the removal of that load.

The results of the two tests are reported separately in Table 16, and they are compared with values obtained from the experimentally-determined influence lines, and also with the values computed by the elastic theory in Table 17. It is of interest to note that at the crown and at the knee the values of the moment determined by the three

TABLE 16
REACTION COMPONENTS DUE TO LIVE LOAD; DESIGN-LOAD TEST
Specimen 1

Load Change	Change in Horizontal Thrust, lb.			Change in Vertical Reaction, lb.						Change in Moment 1000 ft. lb.	
				East Base			West Base				
	East Base	West Base	Average	Outside Scale	Inside Scale	Total	Outside Scale	Inside Scale	Total	East Base	West Base
Bases Hinged											
On	+4299	+4315	+4307	......	......	+4803	......	......	+3775	......	......
Off	−4252	−4273	−4263	......	......	−4802	......	......	+3768	......	......
Average			+4285			+4803			+3772		
Bases Fixed											
On	+5342	+5360	+5351	+3564	+1355	+4899	+3432	+223	+3655	+13.2	+19.3
Off	−5689	−5703	−5696	−3924	− 975	−4899	−3814	+155	−3659	−17.0	−23.8
Average			+5524			+4899			+3657	+15.1	+21.5

TABLE 17
REACTIONS DUE TO DESIGN LIVE LOAD; SUMMARY OF RESULTS

Bases	Source of Information	East Base		West Base		Horizontal Thrust	Moment at Crown 1000 ft. lb.	Moment at Knee 1000 ft. lb.		Rotation of Bases 0.0001 rad.		Sway 0.001 in.
		Moment 1000 ft. lb.	Vertical Reaction lb.	Moment 1000 ft. lb.	Vertical Reaction lb.	lb.		East	West	East	West	
Hinged	From live load test	......	4803	......	3772	4285	25.5	−62.3	−62.6	0.74	1.68	11.7
	From experimental influence ordinates	......	4795	......	3825	4240	26.5	−58.9	−58.9	0.56	1.35	8.5
	From theory	......	4830	......	3790	4020	29.4	−60.5	−60.5	....	....	....
Fixed	From live load test	+15.1	4899	+21.5	3657	5524	24.7	−62.3	−56.2	....	....	2.8
	From experimental influence ordinates	+15.6	4930	+23.0	3690	5430	25.9	−62.5	−54.1	....	....	3.4
	From theory	+25.5	4910	+30.4	3710	5920	28.4	−63.0	−57.4	....	....	....

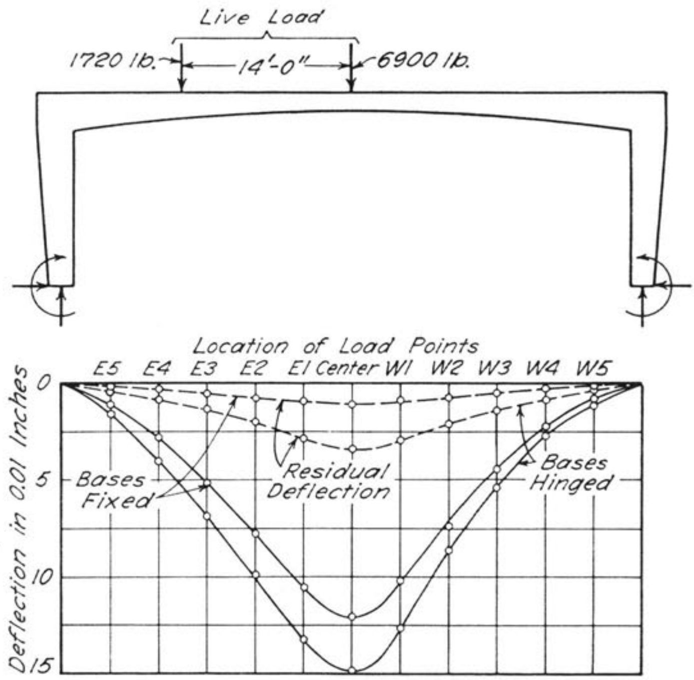


FIG. 24. DEFLECTION OF DECK DUE TO DESIGN LIVE LOAD; SPECIMEN 1

methods, live-load test, experimental influence ordinates, and elastic theory agree fairly well. For the moment at the crown, the value obtained from the design-load test is less than the theoretical value by 15 per cent for the structure with hinged bases, and by 14 per cent for the structure with fixed bases. For the moment at the knee, the value determined from the live-load test exceeds the theoretical value by 3 per cent for the structure with hinged bases, and the two values are in almost perfect agreement for the structure with fixed bases. For the moment at the bases, the values determined by the live-load test are 66 per cent of the theoretical value.

The deflection of the deck is shown in Fig. 24. The full lines represent the deflection with the live load on, and the broken lines the deflection that remained after the live load had been removed.

16. *Capacity Test, Specimen 1.*—As a final test, Specimen 1 was loaded to destruction with the bases hinged. Each increment of load was "one live load" and consisted of 6900 lb. at the crown and 1720 lb. 14 ft. east of the crown. The bases were returned to their normal

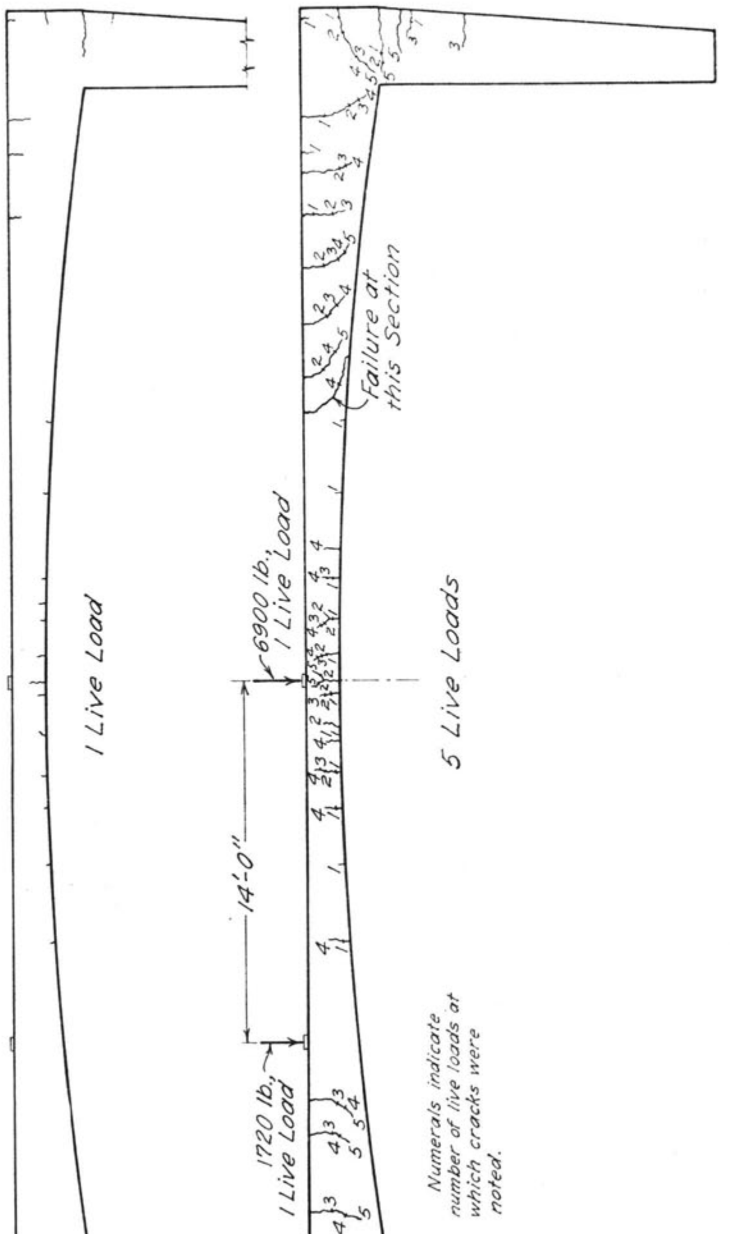


FIG. 25. CRACKS OBSERVED DURING CAPACITY TEST; SPECIMEN 1

TABLE 18
REACTION COMPONENTS AT BASES; CAPACITY TEST
Specimen 1

Load	Vertical Reaction, lb.		Total Live Load	Horizontal Thrust
	East Base	West Base	lb.	lb.
D.L.	20 670	20 670	......	6 087
D.L. + 1 L.L.	25 450	24 448	8 558	10 379
D.L. + 2 L.L.	30 242	28 219	17 121	14 367
D.L. + 3 L.L.	34 988	31 946	25 594	18 307
D.L. + 4 L.L.	39 755	35 704	34 119	21 970
D.L. + 5 L.L.	44 454	39 434	42 548	25 950

positions relative to each other after each load change and the same readings were taken as for the design-load test described in Section 15. The structure was loaded by adding blocks of known weight to the loading platforms until the desired load was obtained. Before adding a load increment, jacks placed under each loading platform were extended until they were in contact with it. All the blocks required for the load increment were then placed on the platforms, and next the jacks were lowered, thus applying the load quickly but gently, and without causing the platform to swing. The bases were adjusted for position and a complete set of readings was taken after each increment up to and including 5 live loads. The structure carried 5 live loads for a period of an hour, but the deck was badly cracked so that the next increment, instead of being one live load, consisted of 2000 lb. at the crown. While the jacks were being lowered failure occurred near the one-quarter point, as indicated on Figs. 25 and 29.

The deflection of the deck after each increment of load is shown in Fig. 26. The reaction components at the bases are given in Table 18. The relation between the live load and the moment is shown in Fig. 27, the moment at the crown being shown in the upper and that at the knee in the lower part of the figure. The light broken lines represent the moments computed from the reaction components measured during the capacity test, and the light full lines the values computed by the elastic theory. The values of the moments computed by the elastic theory agree very closely with the values determined during the capacity test throughout the range of the test. This is true even though the structure was badly cracked.

The position of the thrust line for various loads is shown in Fig. 28. For the live loads not shown, the thrust line falls between the live-load thrust lines that are shown for that portion where the latter cross the axis of the deck. It appears from this figure, therefore, that the point

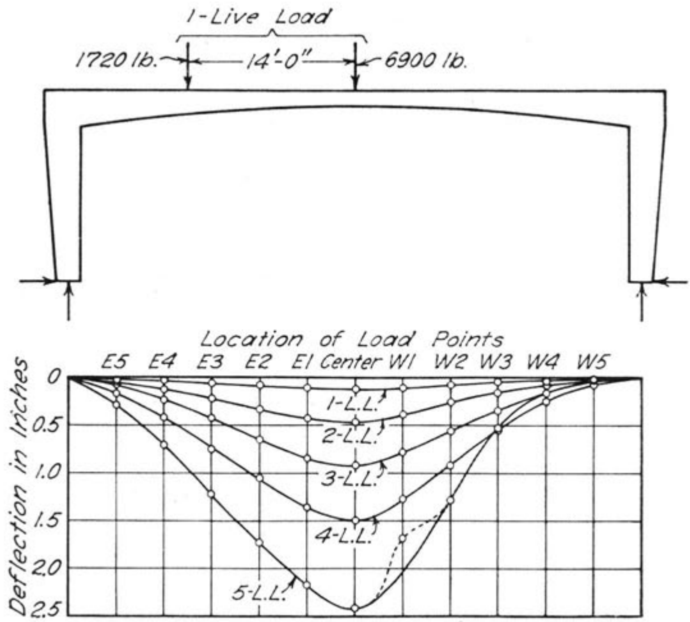


FIG. 26. DEFLECTION OF DECK DUE TO LIVE LOADS; SPECIMEN 1

of contraflexure is confined within narrow limits for the load changes used in the capacity test.

The failure was very abrupt and, as shown in Fig. 29, was due to stripping the longitudinal reinforcing out of the concrete because of the shear at a section where there was a crack and but little moment. The position of the fracture relative to the thrust lines is shown in Fig. 28. The maximum unit shear due to design load was low, 32 lb. per sq. in., and no shear reinforcement was provided. Even at the load that produced failure the unit shear was only 72 lb. per sq. in. at the section of failure. But, from Fig. 25, it is apparent that the shear resistance was greatly reduced by a crack at a section where the moment was not large enough to produce a compression great enough to cause the shear to be resisted by friction. Although the unit shear was so small that specifications do not require the use of shear reinforcement, the structure collapsed because of shear at a section having a flexure crack. The steel reinforcement on the lower side of the deck at the crown was stressed beyond the yield point, as indicated by the strain-gage readings, but spalling had not occurred on the top side of the deck. The shearing stress for highway loading is, in general, low

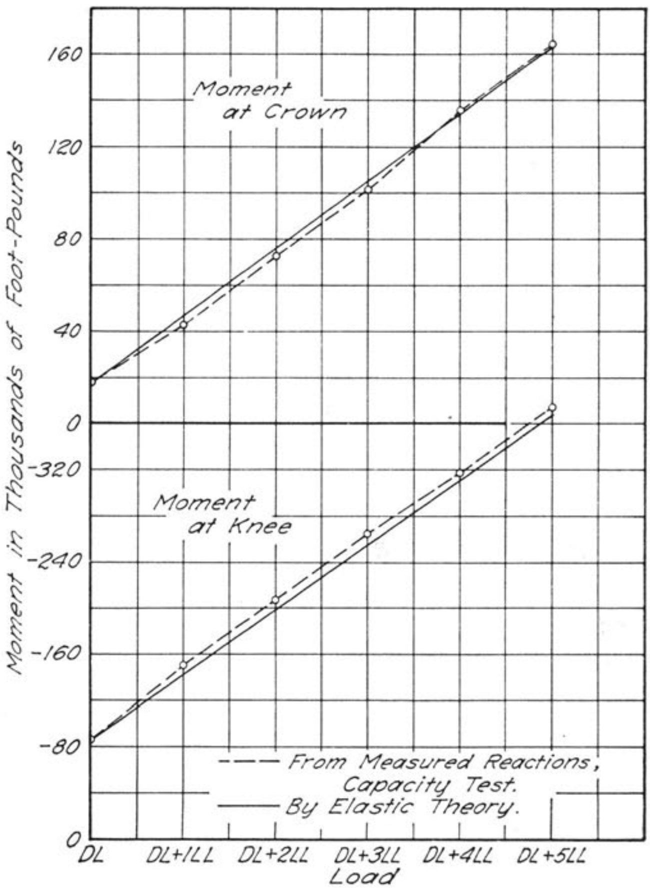


FIG. 27. RELATION BETWEEN LOAD AND MOMENT; CAPACITY TEST; SPECIMEN 1

for this type of structure; however, it would appear from the character of the failure of this structure and the abruptness with which failure occurred that shear reinforcement would be beneficial in reinforced concrete rigid frame bridges even though the shear is small.

The unit compressive stress at the crown and at the knees, computed from the measured thrust and moment by the usual methods for rectangular sections reinforced top and bottom and subject to bending and direct stress was 3100 and 1630 lb. per sq. in., respectively. The unit stresses thus determined are of interest although they are somewhat in error. It is evident that the stresses at the knees were

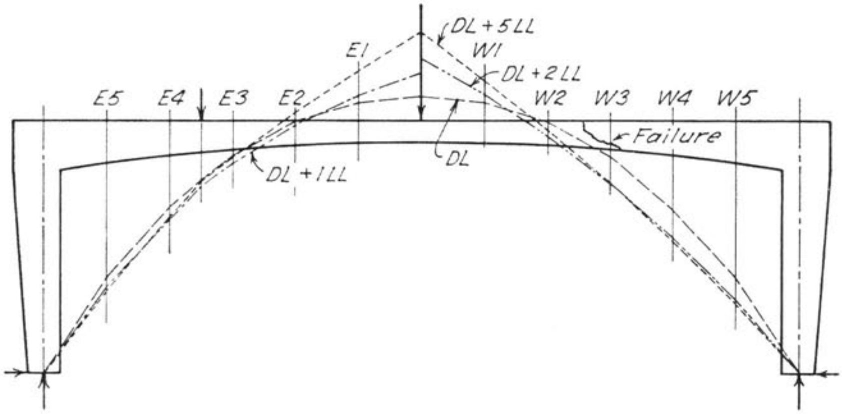


FIG. 28. POSITION OF THRUST LINES FOR SPECIMEN 1; CAPACITY TEST

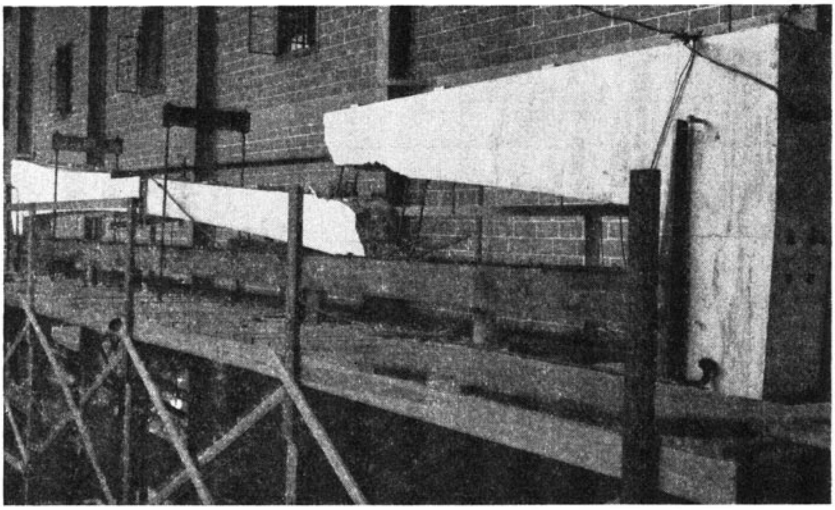


FIG. 29. SPECIMEN 1 AFTER FAILURE

much higher than the computed values indicated, because some of the tension cracks extended to within 3.5 inches of the inside face. There was no evidence of spalling or impending flexural failure, however, at either the crown or the knees. Standard 6 in. x 12 in. control cylinders developed an average ultimate strength of 3980 lb. per sq. in., as given in Table 1.

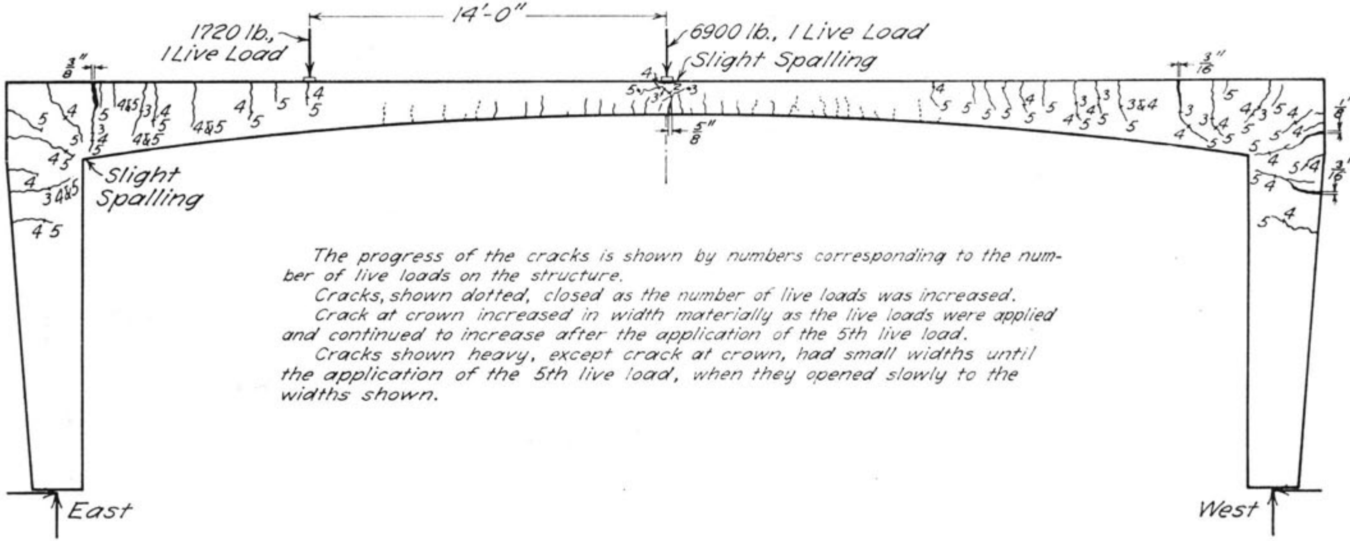


FIG. 30. CRACKS IN SPECIMEN 2; CAPACITY TEST

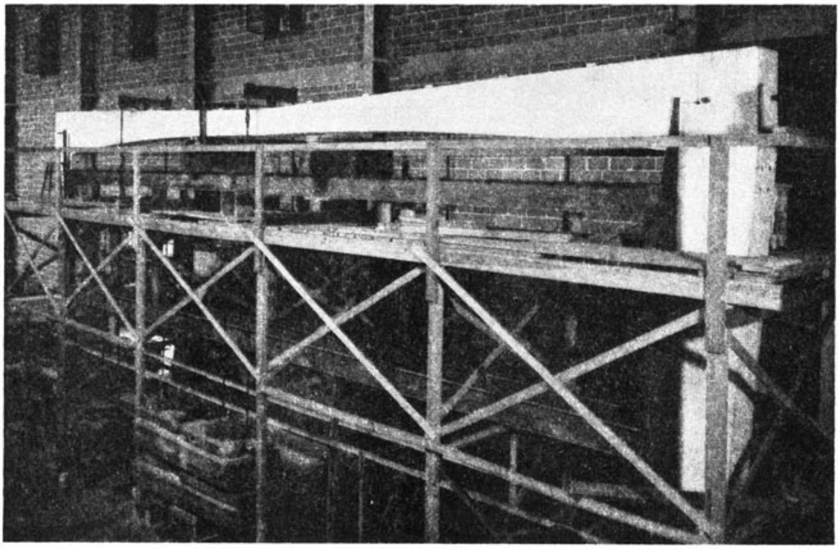


FIG. 31. SPECIMEN 2 WITH BASES SPREAD 4 IN. AND CARRYING FIVE LIVE LOADS

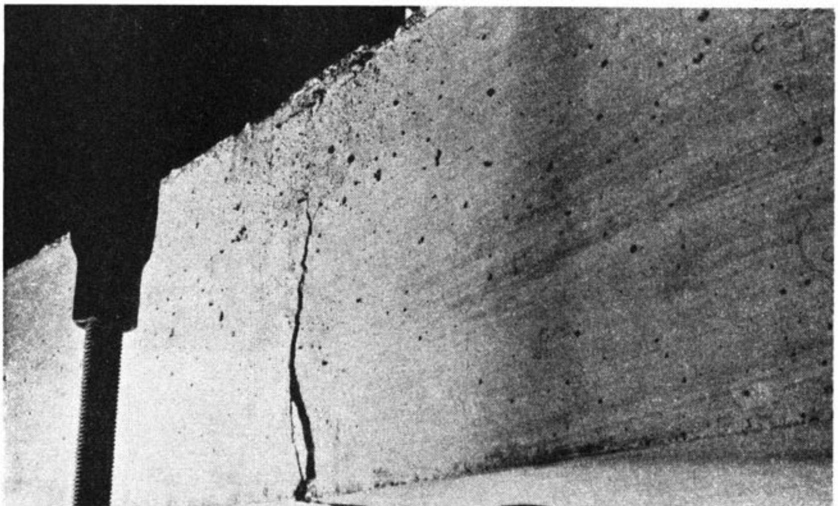
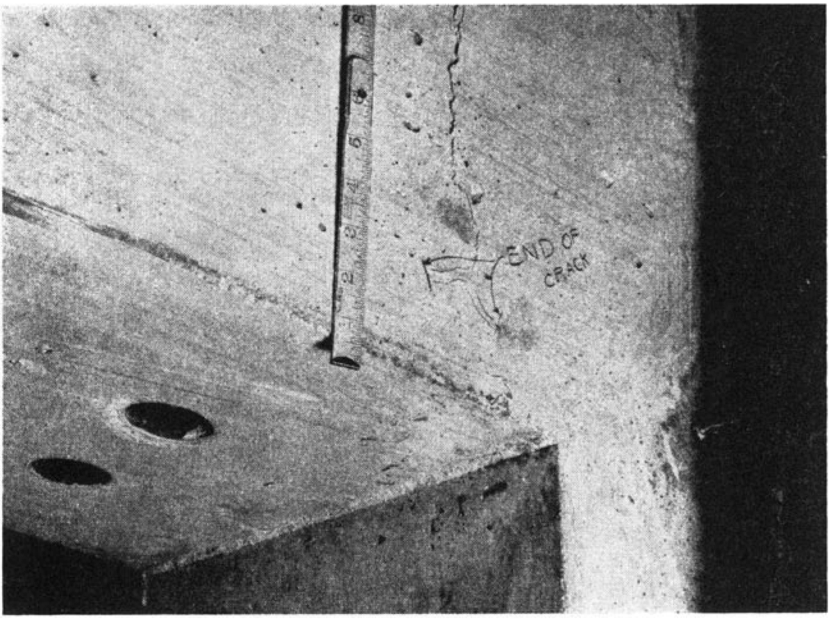
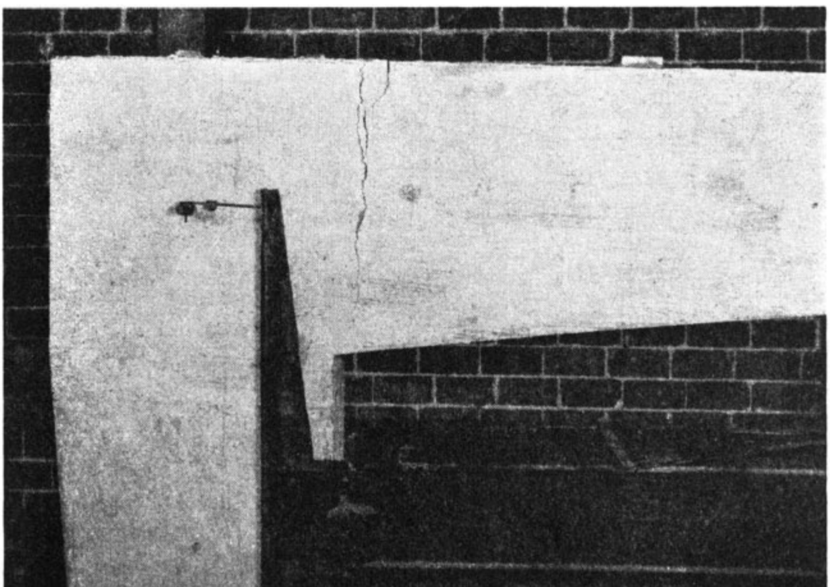


FIG. 32. CRACKS AT CENTER OF SPECIMEN 2 WITH BASES SPREAD 4 IN. AND CARRYING FIVE LIVE LOADS



(a)



(b)

FIG. 33. CRACKS AT EAST KNEE OF SPECIMEN 2 WITH BASES SPREAD 4 IN. AND CARRYING FIVE LIVE LOADS

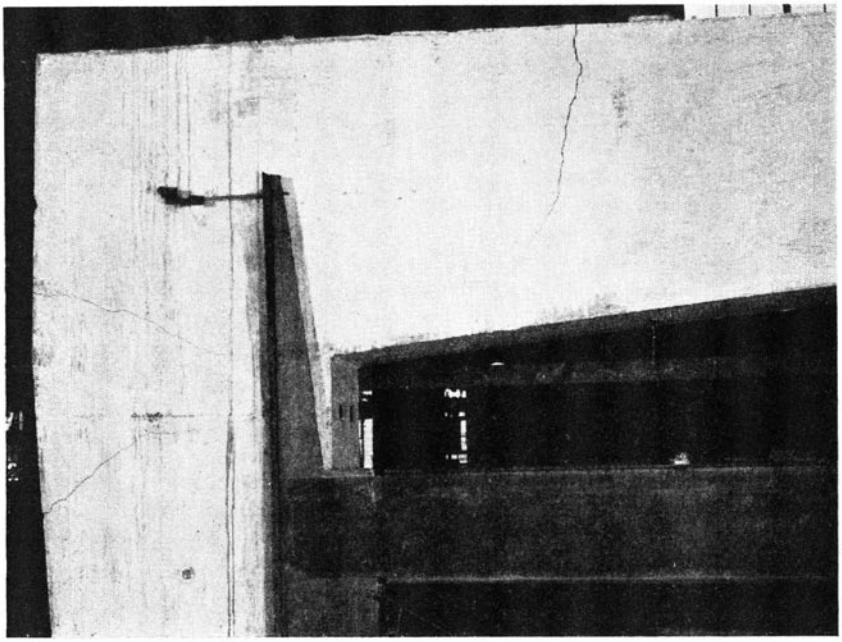


FIG. 34. CRACKS AT WEST KNEE OF SPECIMEN 2 WITH BASES SPREAD 4 IN. AND CARRYING FIVE LIVE LOADS

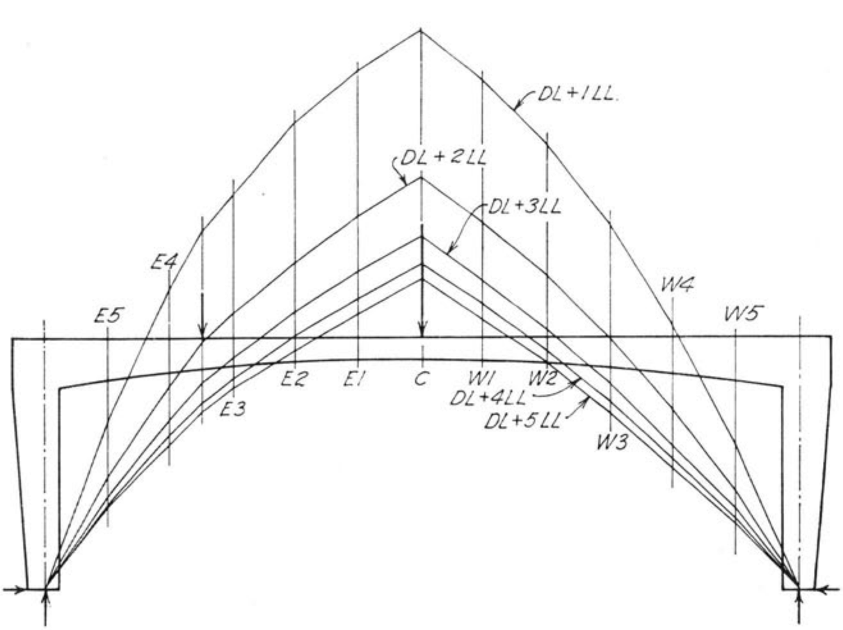


FIG. 35. POSITION OF THRUST LINES FOR SPECIMEN 2; CAPACITY TEST

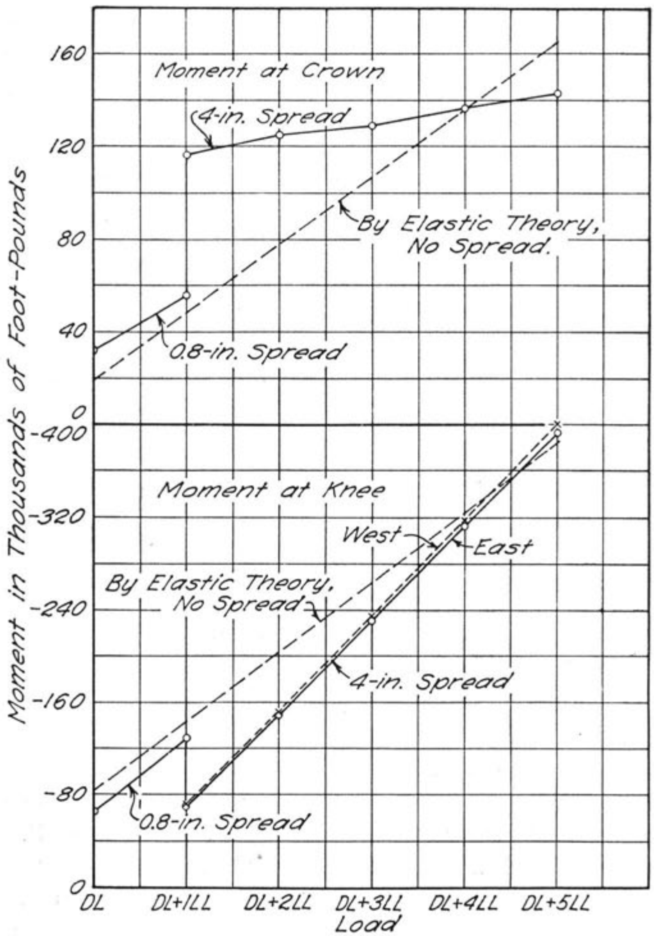


FIG. 36. RELATION BETWEEN LOAD AND MOMENT; CAPACITY TEST; SPECIMEN 2

17. *Capacity Test, Specimen 2.*—After Specimen 2 had been subjected to the test described in Section 14 it was loaded to destruction by increasing the load in increments of one live load. The bases were hinged for this test, and the span was maintained constant at a value 4 in. greater than normal. The method of loading was the same as that used in the capacity test of Specimen 1, described in Section 16.

The principal object of this test was to determine the effect of the 4 in. spread of the bases upon the load-carrying capacity of the structure, and also to determine whether or not the shear reinforcement would make the failure less abrupt than the failure of Specimen 1, which did not have shear reinforcement. The question of the combined effect of live load and spread is seen to be of considerable importance when it is considered that, at the crown, the moment due to spread and the moment due to live load are of the same sign.

The structure was subjected to a maximum of 5 live loads. There was no evidence of serious structural damage when the specimen carried 4 live loads except that the crack at the crown had become wide, indicating that the yield point of the steel had been exceeded. With the application of the fifth live load the steel near the knees was stressed well beyond the yield point, and large cracks opened in the concrete. The yielding at the knees caused a large increase in the deflection of the deck and this deflection was still slowly increasing when the reactions were measured after the fifth live load had been on the structure for more than an hour. It is probable that the five live loads would finally have broken the structure if they had been allowed to remain, as a slight spalling had occurred at the east knee and at the crown when the test was ended. Figure 30 shows the cracks and Fig. 31 the deflection of the structure with the bases spread 4 in. and carrying five live loads. Figures 32 and 33 show the cracks and the spalling at the center of the span and at the east knee, and Fig. 34 shows the cracks at the west knee and also the absence of spalling at this point. Figure 35 shows the position of the thrust line at the various loads.

The relation between the load and the moment is shown in Fig. 36, the upper diagram showing the moment at the crown and the lower one the moment at the knee. The left-hand portion of the two diagrams is based upon the spread test described in Section 14.

The broken lines of Fig. 36 represent the relation between the load and the moment for a structure without spread as determined by the elastic theory. It is of interest to note that, for the moment at the knees, the measured value for the structure carrying 5 live loads and with a spread of 4 in. is only slightly greater than the computed value for a structure carrying 5 live loads but without spread. The moment at the crown has been reduced to its ability to resist and the moment in excess of this ability has been automatically transferred to the knees. The large deviation in the dimensions of the tested structure from those of the structure analyzed accounts for some of the difference between the measured and the computed values of the moments. Figure 37 shows that increasing the load from one live load to four live loads increased the deflection approximately 2 in.

The relation between the load and the strain in the reinforcing

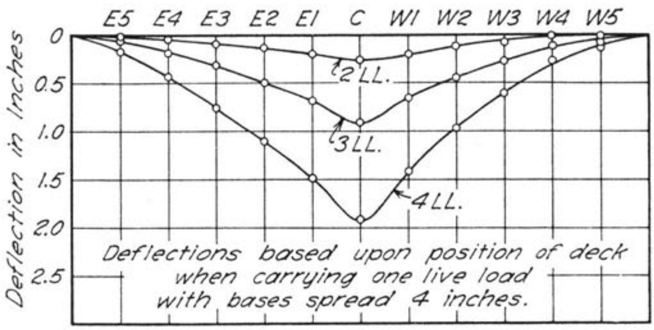


FIG. 37. DEFLECTION OF DECK DUE TO LIVE LOAD; CAPACITY TEST; SPECIMEN 2

steel at the knees and the crown is shown in Fig. 38. The strain shown is the increment over the strain when the structure, with the bases spread 4 in., carried the dead load and one live load. Because of the large increase in the strain in the bottom rods at the crown with the application of the second live load, and because of the wide crack at this point due to spread, it is apparent that the steel at this point was stressed beyond the yield point before the capacity test was started. For the steel at the knees, the increment of strain corresponding to the application of the second live load was very small, and it is probable that the strain at these points at the beginning of the capacity test was also small. The increment in strain at these latter points did not equal the elastic strain corresponding to a yield-point stress until the fourth live load was applied to the structure.

This test indicates that the 4 in. spread of the bases had practically no effect upon the load-carrying capacity of the structure. In order to understand why this should be true it is necessary to distinguish between load stresses and deformation stresses, the latter being stresses due to the change in shape of the structural member.

Figure 39 shows a typical stress–strain diagram for concrete. At a design stress of 1200 lb. per sq. in. the tangent modulus is of the order of 3 000 000 lb. per sq. in., but at stresses near the ultimate it is almost zero. In designing concrete structures, the unit strain due to a deformation such as that produced by temperature changes, shrinkage, or foundation movements is computed from the elastic properties of the member, and the unit strain is converted into stress on the basis of a stress–strain relation corresponding to a design stress. The resulting stress, which is a deformation stress, is then combined with the load stress on the basis that the two are additive.

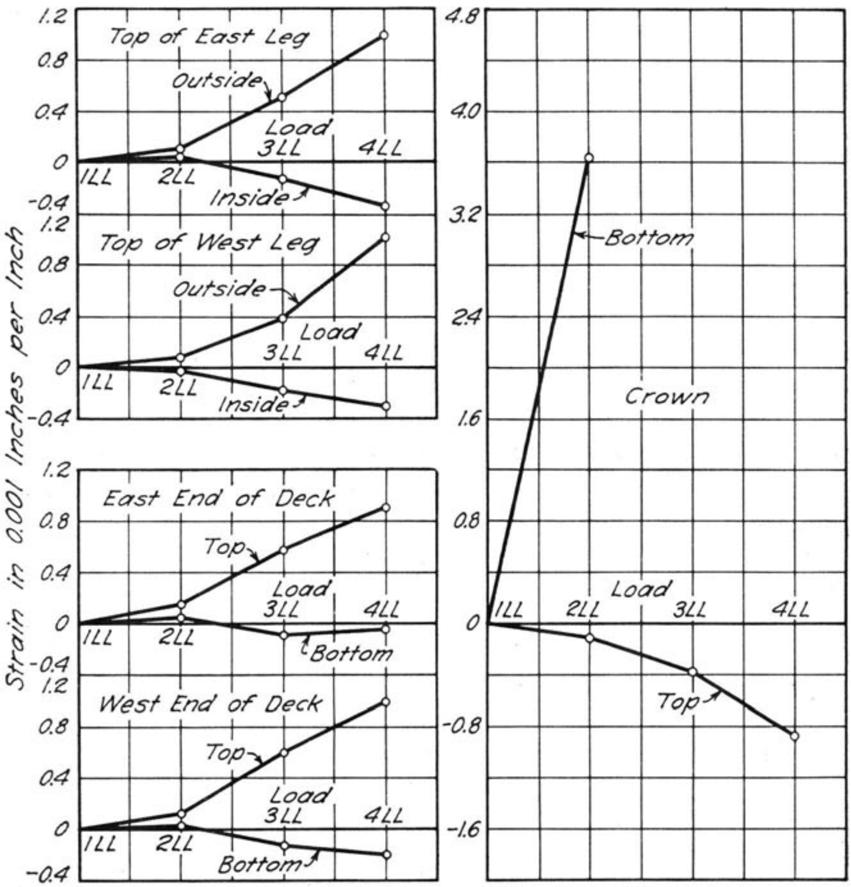


FIG. 38. STRAIN IN REINFORCING STEEL; CAPACITY TEST; SPECIMEN 2

This is proper if the object is to determine the maximum possible stress due to the combination of conditions for which the structure is designed. But if the object is to determine the load-carrying capacity of the structure the method leads to erroneous conclusions since, when the stress is near the ultimate, the strain resulting from the deformation can take place without any appreciable increase in stress and, as the tests show, the deformation stresses do not appreciably affect the load-carrying capacity of the structure.

Another way of looking at the action of the structure leads to the same conclusion. If a rigid frame with hinges at the bases also had a hinge at the crown it would be able to carry loads on the deck but the

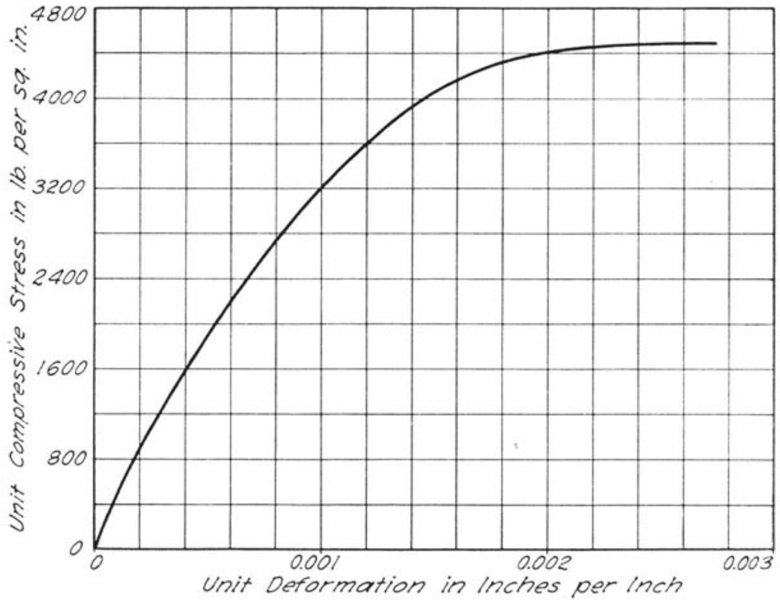


FIG. 39. TYPICAL STRESS–STRAIN DIAGRAM FOR CONCRETE

moment at the knees would be somewhat increased because of the hinge at the crown. But, as rigid frame bridges are usually designed, the moment-resisting capacity is so much less for the section at the crown than for the section at the knees that the increase in moment at the knees due to the introduction of the hinge at the crown is relatively small. When the moment at the crown of Specimen 2 became so large during the capacity test that the stress in the steel was equal to the yield point stress, the action of the crown was similar to that of a moment-resisting hinge. This is apparent from the nearly horizontal portion of the upper curve of Fig. 36. The addition of successive live loads caused some rotation at the crown but, because the steel was stressed beyond the yield point and, later, the concrete began to spall, this rotation did not produce any appreciable increase in the moment. Instead, the moment at the knees increased faster with the load than it would if the moment-resisting capacity of the crown had not been impaired. This action continued until the moment-resisting capacity of the knees had been exceeded, additional live loads causing the gradual collapse of the structure.

It is interesting to note that the shear reinforcement in the deck and legs of the structure not only prevented the abrupt shear failure such as was experienced with Specimen 1, but also gave the structure a tenacity and flexibility greater than that which a reinforced concrete structure is usually considered to possess.

The unit compressive stress computed from the measured reactions, when the bases were spread 4 inches and the structure carried 5 live loads, was 2730 lb. per sq. in. at the crown, and 1720 lb. per sq. in. at the knees. These computed values, although of interest, are obviously in error because the unit stress in the steel reinforcement at these sections was beyond the yield point before the fifth live load was applied to the structure; therefore the concrete stresses were actually higher than the computed values indicate. At the knees the error in the computed stress is even greater because of the unusual distribution of stress in the vicinity of the re-entrant angle.

The average unit strength of the control cylinders was 3114 lb. per sq. in.

IV. Interpretation of Results

18. *Interpretation of Results.*—The tests reported in this bulletin appear to support the following recommendations relative to design.

(1) An analysis of a reinforced concrete rigid frame bridge by the elastic theory gives values for the moment, thrust, and shear on any section which are accurate enough for purposes of design if the analysis is based on the following assumptions:

(a) The stress–strain relation for the concrete has the same value at all sections and at all stresses.

(b) The moment of inertia is for an uncracked section.

(2) The variations in the modulus of elasticity of the concrete that may be expected in a field structure will not have an appreciable effect upon the stresses due to loads.

(3) Restraining the deck of a rigid frame bridge so as to prevent longitudinal sway due to eccentric loads on the deck does not increase the maximum live-load moment at the crown, but does increase the live-load moment at the knee somewhat. But, since the dead load causes no sway and the dead-load moment at the knee is greater than the live-load moment, the resultant moment is not greatly affected. Provision should be made to prevent the structure from being subjected to an active longitudinal horizontal force at the end of the deck due to an expanding road slab or other similar cause.

(4) For a rigid frame bridge of the type tested, a flexural failure at the knee will cause the structure to collapse; a flexural failure at the crown may injure the roadway, but so long as the deck retains its

ability to resist shear and thrust, the structure will not collapse nor will the moment be greatly affected at other sections; a flexural failure at the base will not appreciably affect the moment due to load at other sections, nor will it cause the structure to collapse if the base retains its capacity to resist shear and thrust; a small increase in thickness at the knee will result in a considerable increase in the flexural strength of the knee. For these reasons an approximate determination of the moments at the crown and bases is satisfactory for purposes of design, but it is highly desirable to make ample provisions to resist the shear at these points. Because the moment at the knee is affected by the restraint against sway and is therefore somewhat uncertain, because extra flexural strength of the knee can be obtained with so little cost, and because a flexural failure of the knee is so serious, it is good engineering sense to design the knee for a moment somewhat greater than the moment computed by the elastic theory for structures free to sway.

(5) Variations in the angular restraint of the bases does not appreciably affect the resultant moments (resultant of the dead-load, live-load, temperature, and shrinkage moments) at the knee and crown. But because, for a structure of a given height, the moments at the bases due to shrinkage and temperature changes increase with the span and become excessive for long spans, hinged bases are definitely advantageous for long spans and are not disadvantageous, except possibly for cost, for short spans.

(6) Shear reinforcement added to the tenacity of the reinforced concrete rigid frame bridges tested, thereby increasing the deformation to which they could be subjected without failure.

(7) Deformation stresses of considerable magnitude have no great effect upon the load-carrying capacity of a concrete member properly reinforced for longitudinal and shearing stresses.

This page is intentionally blank.

RECENT PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION†

Bulletin No. 280. The Effect of Residual Longitudinal Stresses upon the Load-Carrying Capacity of Steel Columns, by Wilbur M. Wilson and Rex L. Brown. 1935. *Thirty cents.*

Circular No. 24. Simplified Computation of Vertical Pressures in Elastic Foundations, by Nathan M. Newmark. 1935. *Twenty-five cents.*

Reprint No. 3. Chemical Engineering Problems, by Donald B. Keyes. 1935. *Fifteen cents.*

Reprint No. 4. Progress Report of the Joint Investigation of Fissures in Railroad Rails, by Herbert F. Moore. 1935. *None available.*

Circular No. 25. Papers Presented at the Twenty-second Annual Conference on Highway Engineering, Held at the University of Illinois, Feb. 21 and 22, 1935. 1936. *Fifty cents.*

Reprint No. 5. Essentials of Air Conditioning, by Maurice K. Fahnestock. 1936. *Fifteen cents.*

Bulletin No. 281. An Investigation of the Durability of Molding Sands, by Carl H. Casberg and Carl E. Schubert. 1936. *Sixty cents.*

Bulletin No. 282. The Cause and Prevention of Steam Turbine Blade Deposits, by Frederick G. Straub. 1936. *Fifty-five cents.*

Bulletin No. 283. A Study of the Reactions of Various Inorganic and Organic Salts in Preventing Scale in Steam Boilers, by Frederick G. Straub. 1936. *One dollar.*

Bulletin No. 284. Oxidation and Loss of Weight of Clay Bodies During Firing, by William R. Morgan. 1936. *Fifty cents.*

Bulletin No. 285. Possible Recovery of Coal from Waste at Illinois Mines, by Cloyde M. Smith and David R. Mitchell. 1936. *Fifty cents.*

Bulletin No. 286. Analysis of Flow in Networks of Conduits or Conductors, by Hardy Cross. 1936. *Thirty-five cents.*

Circular No. 26. Papers Presented at the First Annual Conference on Air Conditioning, Held at the University of Illinois, May 4 and 5, 1936. *Fifty cents.*

Reprint No. 6. Electro-Organic Chemical Preparations, by S. Swann, Jr. 1936. *Thirty-five cents.*

Reprint No. 7. Papers Presented at the Second Annual Short Course in Coal Utilization, Held at the University of Illinois, June 11, 12, and 13, 1935. 1936. *None available.*

Bulletin No. 287. The Biologic Digestion of Garbage with Sewage Sludge, by Harold E. Babbitt, Benn J. Leland, and Fenner H. Whitley, Jr. 1936. *One dollar.*

Reprint No. 8. Second Progress Report of the Joint Investigation of Fissures in Railroad Rails, by Herbert F. Moore. 1936. *Fifteen cents.*

Reprint No. 9. Correlation Between Metallography and Mechanical Testing, by Herbert F. Moore. 1936. *Twenty cents.*

Circular No. 27. Papers Presented at the Twenty-third Annual Conference on Highway Engineering, Held at the University of Illinois, Feb. 26-28, 1936. 1936. *Fifty cents.*

Bulletin No. 288. An Investigation of Relative Stresses in Solid Spur Gears by the Photoelastic Method, by Paul H. Black. 1936. *Forty cents.*

Bulletin No. 289. The Use of an Elbow in a Pipe Line for Determining the Rate of Flow in the Pipe, by Wallace M. Lansford. 1936. *Forty cents.*

Bulletin No. 290. Investigation of Summer Cooling in the Warm-Air Heating Research Residence, by Alonzo P. Kratz, Maurice K. Fahnestock, and Seichi Konzo. 1937. *One dollar.*

Bulletin No. 291. Flexural Vibrations of Piezoelectric Quartz Bars and Plates, by J. Tykocinski Tykociner and Marion W. Woodruff. 1937. *Forty cents.*

Reprint No. 10. Heat Transfer in Evaporation and Condensation, by Max Jakob. 1937. *Thirty-five cents.*

Circular No. 28. An Investigation of Student Study Lighting, by John O. Kraehenbuehl. 1937. *Forty cents.*

†Copies of the complete list of publications can be obtained without charge by addressing the Engineering Experiment Station, Urbana, Ill.

Circular No. 29. Problems in Building Illumination, by John O. Kraehenbuehl. 1937. *Thirty-five cents.*
Bulletin No. 292. Tests of Steel Columns; Thin Cylindrical Shells; Laced Channels; Angles, by Wilbur M. Wilson. 1937. *Fifty cents.*
Bulletin No. 293. The Combined Effect of Corrosion and Stress Concentration at Holes and Fillets in Steel Specimens Subjected to Reversed Torsional Stresses, by Thomas J. Dolan. 1937. *Fifty cents.*
Bulletin No. 294. Tests of Strength Properties of Chilled Car Wheels, by Frank E. Richart, Rex L. Brown, and Paul G. Jones. 1937. *Eighty-five cents.*
Bulletin No. 295. Tests of Thin Hemispherical Shells Subjected to Internal Hydrostatic Pressure, by Wilbur M. Wilson and Joseph Marin. 1937. *Thirty cents.*
Circular No. 30. Papers Presented at the Twenty-fourth Annual Conference on Highway Engineering, Held at the University of Illinois, March 3-5, 1937. 1937. *None available.*
Reprint No. 11. Third Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1937. *Fifteen cents.*
Bulletin No. 296. Magnitude and Frequency of Floods on Illinois Streams, by George W. Pickels. 1937. *Seventy cents.*
Bulletin No. 297. Ventilation Characteristics of Some Illinois Mines, by Cloyde M. Smith. 1937. *Seventy cents.*
Bulletin No. 298. Resistance to Heat Checking of Chilled Iron Car Wheels, and Strains Developed Under Long-Continued Application of Brake Shoes, by Edward C. Schmidt and Herman J. Schrader. 1937. *Fifty-five cents.*
Bulletin No. 299. Solution of Electrical Networks by Successive Approximations, by Laurence L. Smith. 1937. *Forty-five cents.*
Circular No. 31. Papers Presented at the Short Course in Coal Utilization, Held at the University of Illinois, May 25-27, 1937. 1938. *None available.*
Bulletin No. 300. Pressure Losses Resulting from Changes in Cross-Sectional Area in Air Ducts, by Alonzo P. Kratz and Julian R. Fellows. 1938. *Sixty-five cents.*
Bulletin No. 301. The Friction of Railway Brake Shoes at High Speed and High Pressure, by Herman J. Schrader. 1938. *Sixty cents.*
Bulletin No. 302. Fatigue Tests of Riveted Joints, by Wilbur M. Wilson and Frank P. Thomas. 1938. *One dollar.*
Circular No. 32. Two Investigations on Transit Instruments, by William H. Rayner. 1938. *Twenty-five cents.*
Circular No. 33. Papers Presented at the Twenty-fifth Annual Conference on Highway Engineering, Held at the University of Illinois, March 2-4, 1938. 1938. *None available.*
Bulletin No. 303. Solutions for Certain Rectangular Slabs Continuous Over Flexible Supports, by Vernon P. Jensen. 1938. *One dollar.*
Bulletin No. 304. A Distribution Procedure for the Analysis of Slabs Continuous Over Flexible Beams, by Nathan M. Newmark. 1938. *One dollar.*
Circular No. 34. The Chemical Engineering Unit Process—Oxidation, by Donald B. Keyes. 1938. *Fifty cents.*
Circular No. 35. Factors Involved in Plate Efficiencies for Fractionating Columns, by Donald B. Keyes. 1938. *Twenty cents.*
**Bulletin No. 305.* Summer Cooling in the Warm-Air Heating Research Residence with Cold Water, by Alonzo P. Kratz, Seichi Konzo, Maurice K. Fahnestock and Edwin L. Broderick. 1938. *Ninety cents.*
**Bulletin No. 306.* Investigation of Creep and Fracture of Lead and Lead Alloys for Cable Sheathing, by Herbert F. Moore, Bernard B. Betty, and Curtis W. Dollins. 1938. *One dollar.*
Reprint No. 12. Fourth Progress Report of the Joint Investigation of Fissures in Railroad Rails, by H. F. Moore. 1938. *None available.*
**Bulletin No. 307.* An Investigation of Rigid Frame Bridges: Part I, Tests of Reinforced Concrete Knee Frames and Bakelite Models, by Frank E. Richart, Thomas J. Dolan, and Tilford A. Olson. 1938. *Fifty cents.*
**Bulletin No. 308.* An Investigation of Rigid Frame Bridges: Part II, Laboratory Tests of Reinforced Concrete Rigid Frame Bridges, by W. M. Wilson, R. W. Kluge, and J. V. Coombe. 1938. *Eighty-five cents.*

*A limited number of copies of bulletins starred are available for free distribution.

This page is intentionally blank.

This page is intentionally blank.

UNIVERSITY OF ILLINOIS

Colleges and Schools at Urbana

COLLEGE OF LIBERAL ARTS AND SCIENCES.—General curriculum with majors in the humanities and sciences; specialized curricula in chemistry and chemical engineering; general courses preparatory to the study of law and journalism; pre-professional training in medicine, dentistry, and pharmacy.

COLLEGE OF COMMERCE AND BUSINESS ADMINISTRATION.—Curricula in general business, trade and civic secretarial service, banking and finance, insurance, accountancy, transportation, commercial teaching, foreign commerce, industrial administration, public utilities, and commerce and law.

COLLEGE OF ENGINEERING.—Curricula in agricultural engineering, ceramics, ceramic engineering, chemical engineering, civil engineering, electrical engineering, engineering physics, general engineering, mechanical engineering, metallurgical engineering, mining engineering, and railway engineering.

COLLEGE OF AGRICULTURE.—Curricula in agriculture, floriculture, general home economics, and nutrition and dietetics.

COLLEGE OF EDUCATION.—Curricula in education, agricultural education, home economics education, and industrial education. The University High School is the practice school of the College of Education.

COLLEGE OF FINE AND APPLIED ARTS.—Curricula in architecture, art, landscape architecture, and music.

COLLEGE OF LAW.—Professional curriculum in law.

SCHOOL OF JOURNALISM.—General and special curricula in journalism.

SCHOOL OF PHYSICAL EDUCATION.—Curricula in physical education for men and for women.

LIBRARY SCHOOL.—Curriculum in library science.

GRADUATE SCHOOL.—Advanced study and research.

University Extension Division.—For a list of correspondence courses conducted by members of the faculties of the colleges and schools at Urbana and equivalent to courses offered to resident students, address the Director of the Division of University Extension, 354 Administration Building, Urbana, Illinois.

Colleges in Chicago

COLLEGE OF MEDICINE.—Professional curriculum in medicine.

COLLEGE OF DENTISTRY.—Professional curriculum in dentistry.

COLLEGE OF PHARMACY.—Professional curriculum in pharmacy.

University Experiment Stations, and Research and Service Bureaus at Urbana

AGRICULTURAL EXPERIMENT STATION
ENGINEERING EXPERIMENT STATION
EXTENSION SERVICE IN AGRICULTURE AND HOME ECONOMICS
BUREAU OF BUSINESS RESEARCH
BUREAU OF COMMUNITY PLANNING
BUREAU OF EDUCATIONAL RESEARCH
BUREAU OF INSTITUTIONAL RESEARCH

State Scientific Surveys and Other Divisions at Urbana

STATE GEOLOGICAL SURVEY
STATE NATURAL HISTORY SURVEY
STATE WATER SURVEY
STATE HISTORICAL SURVEY
STATE DIAGNOSTIC LABORATORY (Animal Pathology)
U. S. SOYBEAN PRODUCTS LABORATORY
U. S. WEATHER BUREAU STATION

For general catalog of the University, special circulars, and other information, address

THE REGISTRAR, UNIVERSITY OF ILLINOIS
URBANA, ILLINOIS